Carrollian and Non-relativistic Jackiw-Teitelboim Supergravity

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Overview

Einstein-Hilbert action in two dimensions

- First guess for two dimensional theory

$$S_{EH} = \frac{1}{16\pi G} \int_M dx^2 \sqrt{-g} R \,,$$

  where R is the Ricci scalar and G is Newton const.

- Topological term.
- $T_{\mu\nu} = 0$ since $G_{\mu\nu} = 0$.

JT gravity

- The Jackiw-Teitelboim gravity action is [1]

$$S = \frac{1}{16\pi G}\int_M dx^2\sqrt{-g}\phi\,(R-2\Lambda)\,,$$

where $\Lambda = -\frac{1}{\ell^2}$ is cosmological constant.

- Equation of motions are

$$\begin{aligned} R-2\Lambda &= 0\,, \\ g_{\mu\nu}\nabla_\sigma\nabla^\sigma\phi - \nabla_\mu\nabla_\nu\phi &= \phi\Lambda g_{\mu\nu}\,. \end{aligned}$$

[1] Teitelboim '83, Jackiw '85, Almheiri, Polchinski, Grumiller.

JT gravity as a BF theory

- JT model as a BF [2] theory.
- BF action is given by

$$I = \frac{k}{2\pi} \int_{M_2} g_{LK} X^L \left(dA^K + \frac{1}{2} C_{IJ}{}^K A^I \wedge A^J \right) .$$

- $C_{IJ}{}^K$ are structure constants and generators e_I are

$$[e_I, e_J] = C_{IJ}{}^K e_K .$$

- Gauge transformations and field equations are

$$\begin{aligned} \delta_\lambda A^I &= -d\lambda^I - C_{JK}{}^I A^J \lambda^K , \qquad \delta_\lambda X_I = C_{IJ}{}^K \lambda^J X_K , \\ F^I &= dA^I + \frac{1}{2} C_{JK}{}^I A^J \wedge A^K = 0 , \quad dX_I + C_{IJ}{}^K A^J X_K = 0 . \end{aligned}$$

[2] Fukuyama,Jackiw

JT gravity as a BF theory

- First-order formulation of JT gravity in BF formulation basically can be constructed by the gauging the AdS_2 algebra

$$[J, P_A] = \epsilon_{AB} P^B \,, \qquad [P_A, P_B] = -\Lambda \epsilon_{AB} J \,,$$

- Lie algebra admits invariant, non-degenerate and symmetric metric as

$$g_{AB} = (P_A, P_B) = \frac{\eta_{AB}}{\ell^2} \,, \quad g_{JJ} = (J, J) = 1 \,.$$

JT gravity as a BF theory

- Gauge fields Lagrange multipliers are axpanded as

$$A = E^A P_A + \Omega J\,, \quad X = X^A P_A + XJ\,, \tag{1}$$

- Covariant curvatures are given by

$$\begin{aligned} R(P)^A &= dE^A - \epsilon^{AB}\Omega \wedge E_B\,, \\ R(J) &= d\Omega + \frac{1}{2\ell^2}\epsilon_{AB}E^A \wedge E^B\,. \end{aligned} \tag{2}$$

- *BF* theory formulation of JT gravity is given by

$$S_{JT} = \frac{k}{2\pi}\int_M \left(-\Lambda X^A R(P)_A + XR(J)\right)\,. \tag{3}$$

JT supergravity as a BF theory

$\mathcal{N} = 2$ AdS_2 superalgebra

$$
\begin{aligned}
[J, P_A] &= \epsilon_{AB} P^B \,, & [P_A, P_B] &= \frac{1}{\ell^2} \epsilon_{AB} J \,, \\
[J, Q^i] &= \frac{1}{2} \gamma_* Q^i \,, & [P_A, Q^i] &= \frac{1}{2\ell} \, \gamma_A Q^i \,, \\
[U, Q^i] &= \frac{1}{2} \epsilon^{ij} Q_j \,,
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
\{Q^i_\alpha, Q^j_\beta\} &= \delta^{ij} (\gamma_A C^{-1})_{\alpha\beta} P^A + \frac{\delta^{ij}}{\ell} \, (\gamma_* C^{-1})_{\alpha\beta} J \,, \\
\{Q^1_\alpha, Q^2_\beta\} &= (C^{-1})_{\alpha\beta} U \,.
\end{aligned}
\tag{5}
$$

JT supergravity as a BF theory

- The invariant bilinear forms are defined as

$$
\begin{aligned}
&(P_A, P_B) = \frac{\eta_{AB}}{\ell^2}\,, \quad (J, J) = 1\,, \quad (U, U) = \frac{1}{\ell}\,,\\
&(Q^i_\alpha, Q^j_\beta) = \frac{2\delta^{ij}}{\ell}(C^{-1})_{\alpha\beta}\,.
\end{aligned} \tag{6}
$$

- Action [3]

$$
S_{SJT} = \frac{k}{2\pi}\int_M \left(\frac{1}{\ell^2}X^A R(E)_A + XR(\Omega) + \frac{1}{\ell}YR(U) + \frac{1}{\ell}\bar{\lambda}^i R(Q_i)\right)\,, \tag{7}
$$

where λ^i are fermionic Lagrange multipliers and $i = 1, 2$ indicates the number of supercharges.

[3] Grumiller

Lie algebra Expansion

- We split the Lorentz indices as $A = (0, 1)$, such that

$$P_A = (P_0, P_1) \; , \quad \gamma^A = \left(\gamma^0, \gamma^1\right) \; . \tag{8}$$

- Introduce the following combination of the fermionic generators Q^i_α:

$$\tilde{Q}^\pm = \frac{1}{\sqrt{2}} \left(Q^1 \pm \gamma_0 Q^2\right) \; . \tag{9}$$

- Then, we consider the following expansion of the generators:

$$\begin{aligned} &P_0 = H + \eta^2 M \, , \quad P_1 = \eta P \, , \quad J = \eta G \, , \quad U = U_1 + \eta^2 U_2 \\ &\tilde{Q}^+ = Q^+ + \eta^2 R \, , \quad \tilde{Q}^- = \eta Q^- \, , \end{aligned} \tag{10}$$

where η is the expansion parameter.

Non-relativistic JT supergravity

- Non-degenerate, invariant metric is given by

$$(P,P)=\frac{1}{\ell^2}\,,\quad (G,G)=1\,,\quad (H,M)=-\frac{1}{\ell^2}\,,\quad (U_1,U_2)=\frac{1}{\ell^2}\,,$$
$$(Q^+_\alpha,R_\beta)=\frac{2}{\ell}\left(C^{-1}\right)_{\alpha\beta}\,,\quad (Q^-_\alpha,Q^-_\beta)=\frac{2}{\ell}\left(C^{-1}\right)_{\alpha\beta}\,.$$

- Gauge fields are

$$A \;=\; \tau H+eP+\omega G+mM+r_1U_1+r_2U_2+\bar{\psi}^+Q^+ +\bar{\psi}^-Q^- +\bar{\rho}R\,.$$

Non-relativistic JT supergravity

- Using invariant metric and curvatures, BF action for NH_2^- JT supergravity is given by [4]

$$S_{\text{sJT}}^{\text{NR}} = \frac{k}{2\pi}\int \Big[\Phi R(\omega) + \frac{1}{\ell^2}\left(\Pi R(e) - \Xi R(m) - \Sigma R(\tau)\right) + \frac{1}{\ell^2}\left(T_1 R(r_2) + T_2 R(r_1)\right) + \frac{2}{\ell}\left(\bar{\lambda}^+ R(\psi^+) + \bar{\lambda}^- R(\psi^-) + \bar{\lambda} R(\rho)\right)\Big]. \tag{11}$$

- This action is supersymmetric generalization of bosonic NRJT action (Grumiller, Hartong, Gomis, Rebolledo).

[4] L. Ravera and U.Z.

Ultra-relativistic JT supergravity

- Ultra-relativistic (Carrollian) limit is $c \to 0$.
- Perform the following redefinition on the sNH_2 structure

$$H \leftrightarrow P\,, \quad \ell \to -i\ell\,,$$
$$\gamma_0 \to -i\gamma_1\,, \quad \gamma_1 \to -i\gamma_0\,, \quad \gamma_\star \to \gamma_\star\,, \quad C \to -iC\,.$$

- We end up with the supersymmetric extended AdS_2 Carroll algebra.
- Duality: the E $NH^{\pm} \to$ E (A)dS_2 Carroll algebras.

Ultra-relativistic JT supergravity

- For expansion, we decompose the fermionic charges as

$$F^{\pm} = \frac{1}{\sqrt{2}} \left(Q^1 \pm i\gamma_1 Q^2 \right) . \tag{12}$$

- Expansion of generators

$$\begin{aligned} &P_0 = \eta H \,, \quad P_1 = P + \eta^2 M \,, \quad J = \eta G \,, \quad U = U_1 + \eta^2 U_2 \\ &F^+ = Q^+ + \eta^2 R \,, \quad F^- = \eta Q^- \,. \end{aligned}$$

- The extended AdS_2 Carroll superalgebra is endowed with the invariant metric

$$\begin{aligned} &\langle H, H \rangle = -\frac{1}{\ell^2} \,, \quad \langle G, G \rangle = 1 \,, \quad \langle P, M \rangle = \frac{1}{\ell^2} \,, \quad \langle U_1, U_2 \rangle = -\frac{1}{\ell^2} \,, \\ &\langle Q^+_\alpha, R_\beta \rangle = \frac{2}{\ell} \left(C^{-1} \right)_{\alpha\beta} \,, \quad \langle Q^-_\alpha, Q^-_\beta \rangle = \frac{2}{\ell} \left(C^{-1} \right)_{\alpha\beta} \,. \end{aligned}$$

Ultra-relativistic JT supergravity

- Ultra-relativistic JT supergravity action as a metric BF theory based on the extended AdS_2 Carroll superalgebra [5]

$$\mathcal{S}^{\text{UR}}_{\text{sJT}} = \frac{k}{2\pi} \int \Big[\Phi R(\omega) + \frac{1}{\ell^2} \left(\Sigma R(e) + \Pi R(m) - \Xi R(\tau) \right) , \\ - \frac{1}{\ell^2} \left(T_1 R(r_2) + T_2 R(r_1) \right) \\ + \frac{2}{\ell} \left(\bar{\lambda}^+ R(\psi^+) + \bar{\lambda}^- R(\psi^-) + \bar{\lambda} R(\rho) \right) \Big] . \tag{13}$$

- This action is supersymmetric generalization of bosonic Carrollian JT action (Grumiller, Hartong, Gomis, Rebolledo).

[5] L. Ravera and U.Z.

Conclusion

- The supersymmetric extensions of non- and ultra-relativistic JT gravity.
- Duality is preserved in the action level.
- One can start directly from the relativistic $\mathcal{N} = 2$ JT supergravity.
- Then, expand the curvatures and dilaton and dilatini fields, accordingly.
- Even and odd structure of expansion and fermionic decomposition determine the ultra or non-relativistic actions/algebras.

Conclusion

- BF theories beyond the supersymmetric Newton-Hooke and AdS Carrollian models.
- The next order boundary Schwarzian actions.
- Supersymmetric extensions of the Carrollian and non-relativistic boundary Schwarzian actions.
- Ultra- and nonrelativistic limits (and associated supersymmetric extensions) of the most general deformation of JT gravity.

Thank you.