STEERING WITNESSES FOR GAUSSIAN STATES

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EPR PAPER

Einstein – Podolski – Rosen Paradox

"Can Quantum-Mechanical description of Physical Reality be Considered Complete?"

(1935)

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle),$$

where $|0\rangle = |z^+\rangle$ and $|1\rangle = |z^-\rangle$ denote the two possible spin orientations in $z-$ direction.

Alice: z direction

Bob's state

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“If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.“

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"In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other.

Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality."

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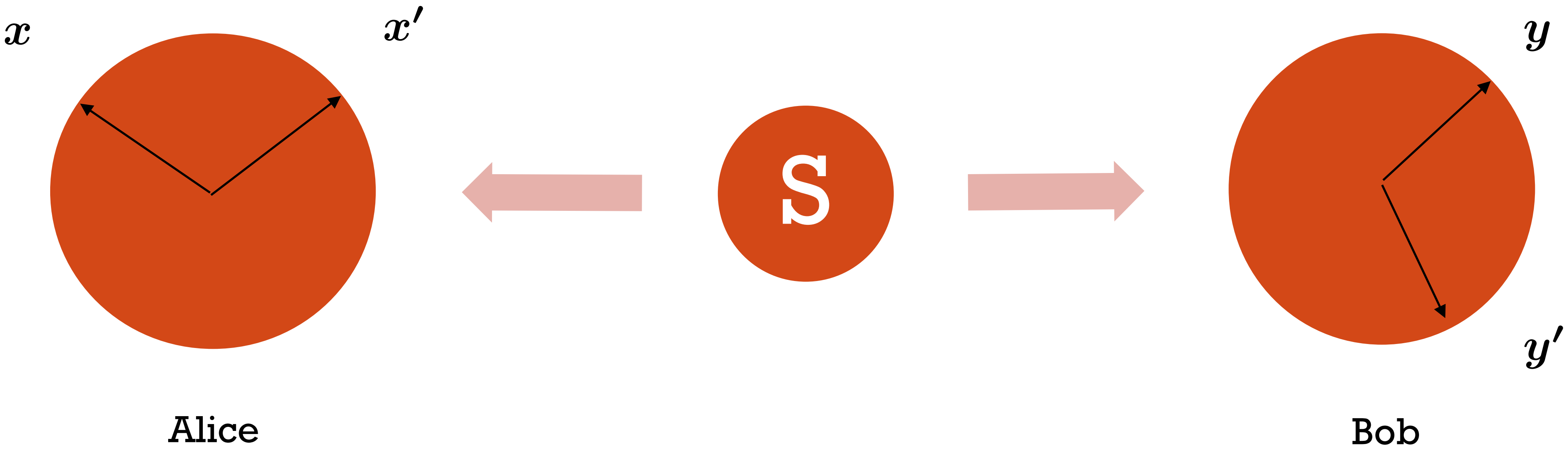
BELL TEST

"On the Einstein Podolski Rosen paradox" (1964)

Local Realism?

Local Hidden Variable?

BELL TEST

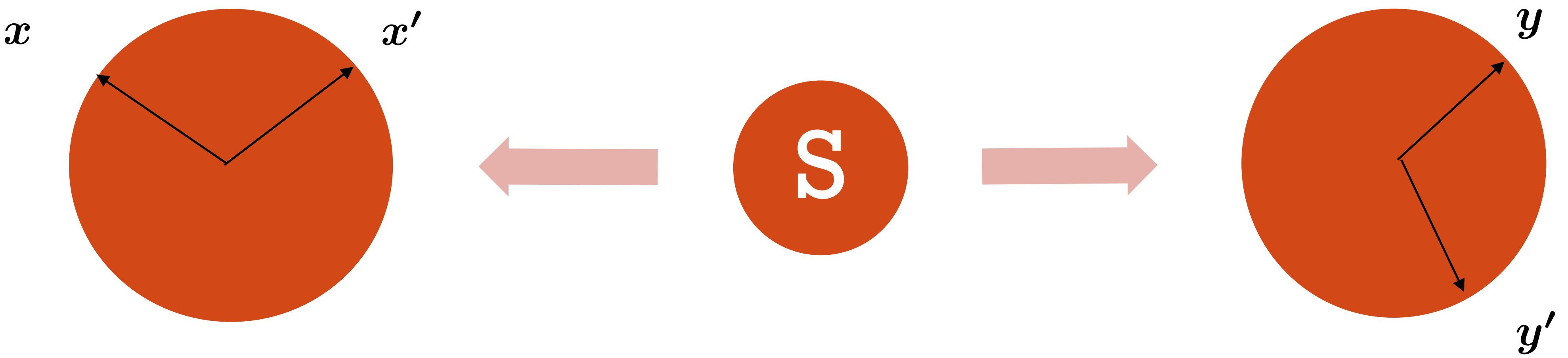


Measurements: $A, \quad a \in \{\pm 1\}$

$A', \quad a' \in \{\pm 1\}$

Measurements: $B, \quad b \in \{\pm 1\}$

$B', \quad b' \in \{\pm 1\}$



Outcomes: $A, \quad a \in \{\pm 1\}$

$A', \quad a' \in \{\pm 1\}$

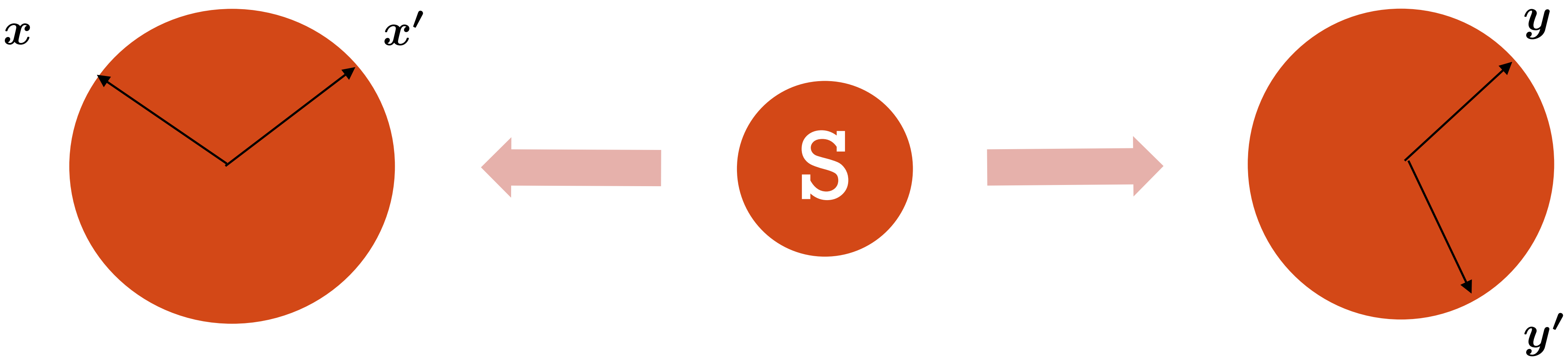
Outcomes: $B, \quad b \in \{\pm 1\}$

$B', \quad b' \in \{\pm 1\}$

a	a'	b	b'
+1	+1	+1	-1
+1	+1	-1	+1
+1	-1	+1	+1
…	…	…	…

Local Realism:

Quantum mechanics:



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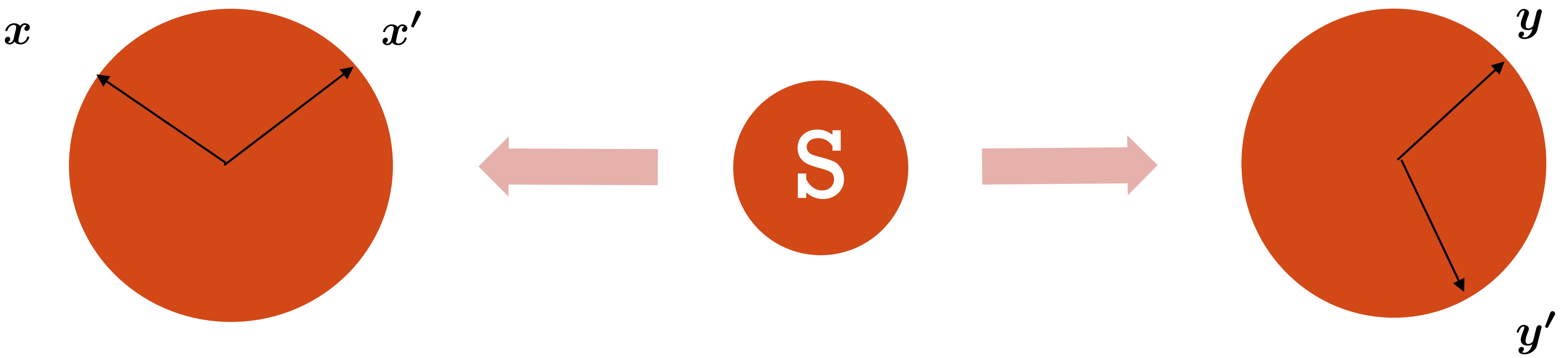
Local Realism:

$$ab + a'b + ab' - a'b' \leq 2$$

Quantum mechanics:



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Quantum mechanics:

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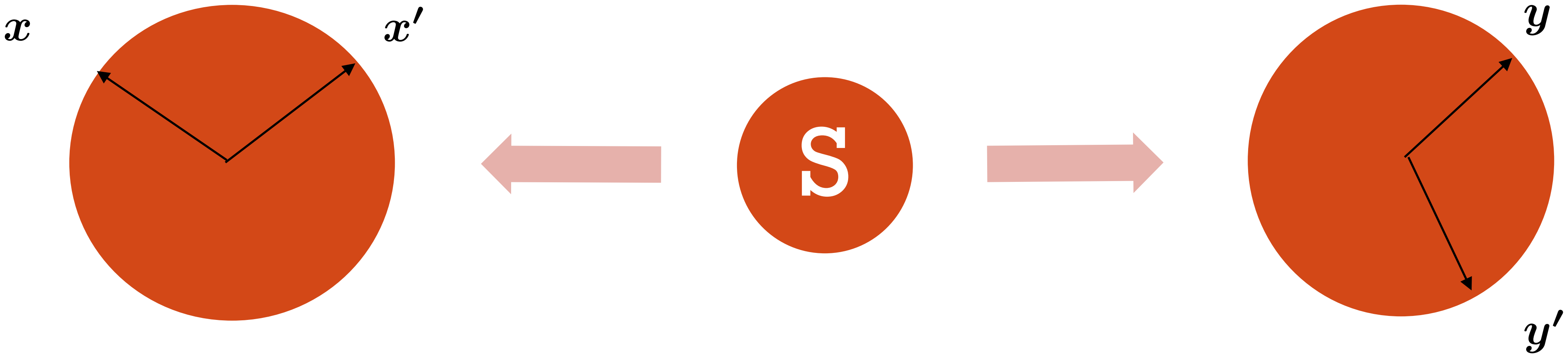

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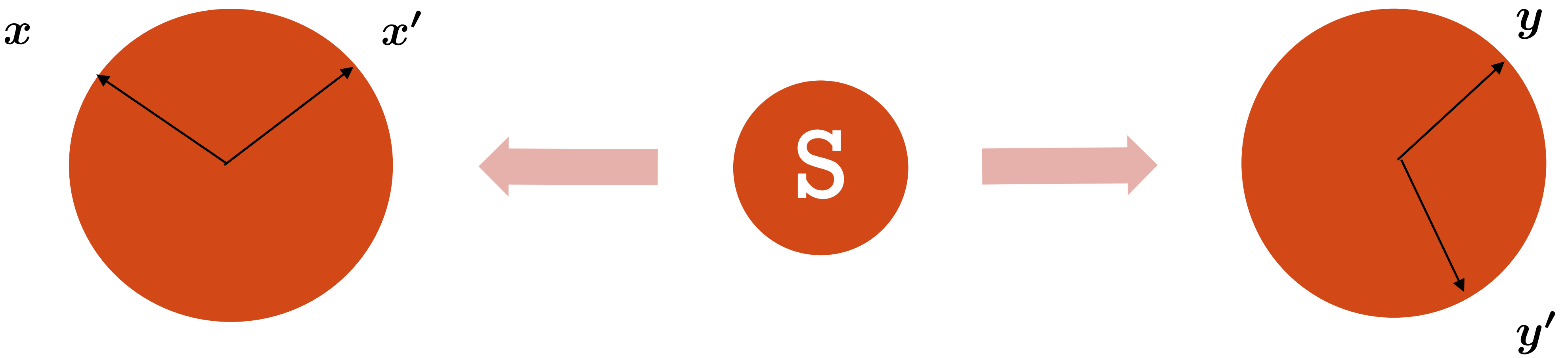

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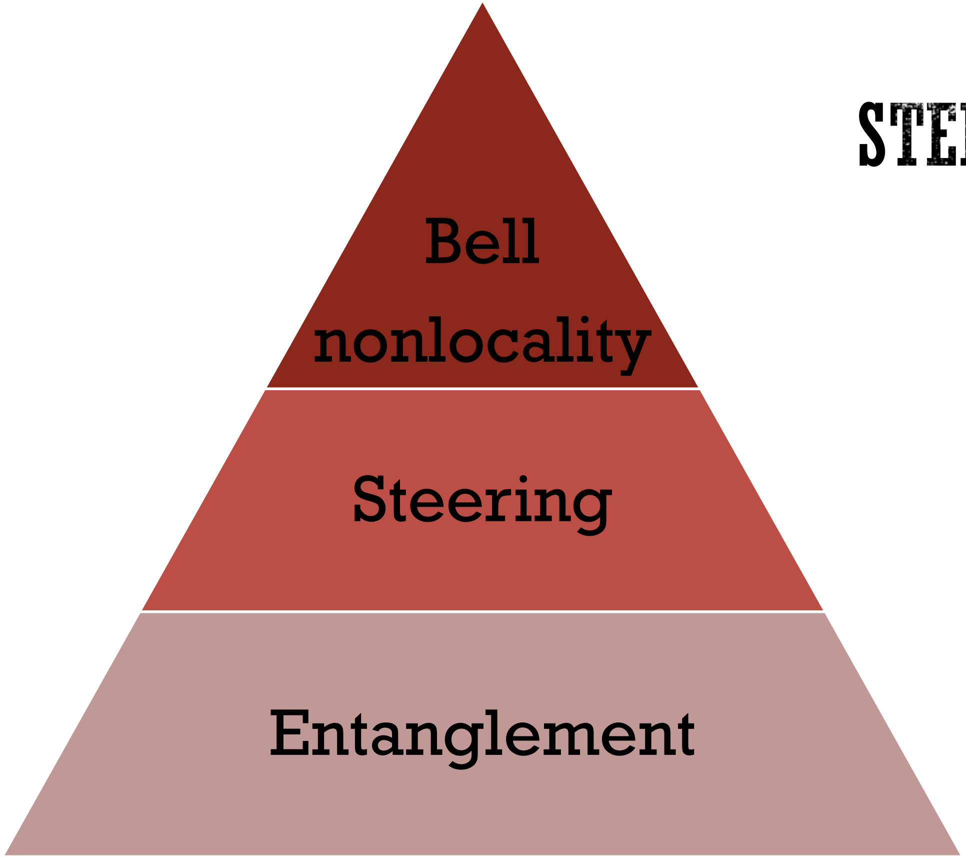
Einstein was wrong!

BELL NONLOCALITY

Failure of Local Hiddenn Variable (LHV) model

$$p(a,b|x,y) = \int d\lambda p(\lambda) p(a|x,\lambda) p(b|y,\lambda)$$



STEERING

Failure of Hybrid LHV – LQS model

ENTANGLEMENT

Failure of Local Quantum State (LQS) model

H. M. Wiseman, S. J. Jones, A. C. Doherty, *Phys.Rev. Lett.* **98**,140402 (2007)

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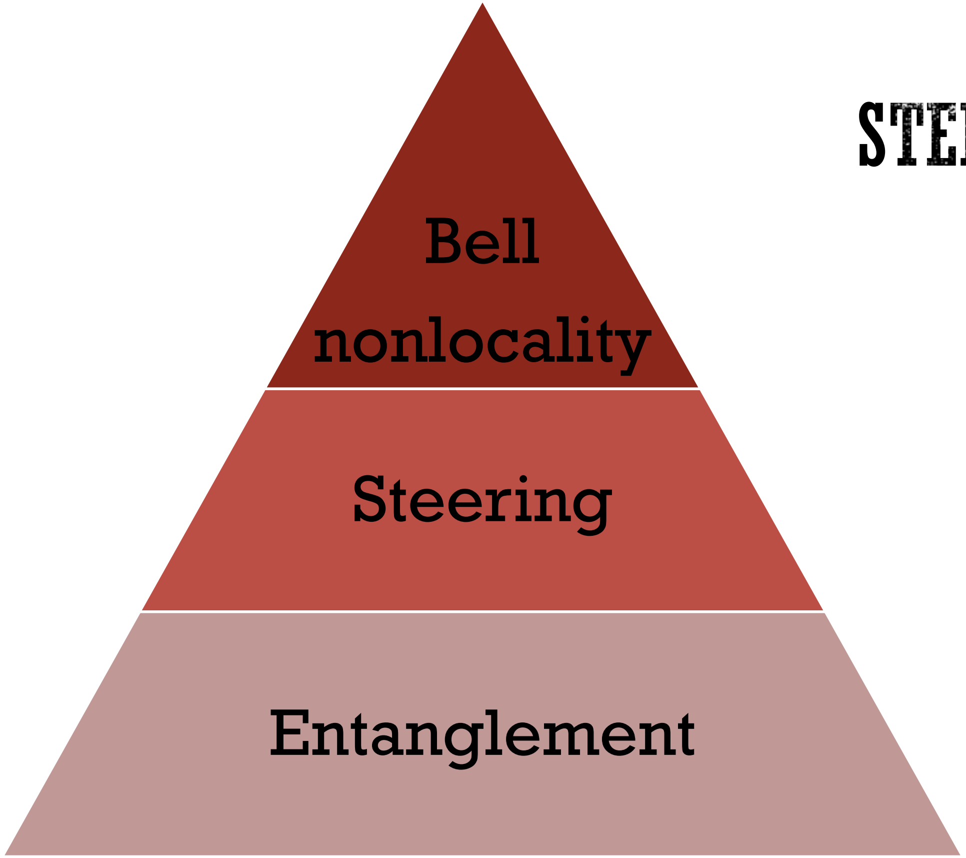

STEERING

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$$p(a,b|x,y) = \sum_k p_k \mathrm{Tr}[E_{a|x}\rho_k^A]\mathrm{Tr}[E_{b|x}\rho_k^B]$$

$$\rho = \sum_k \rho_k^A \otimes \rho_k^B$$

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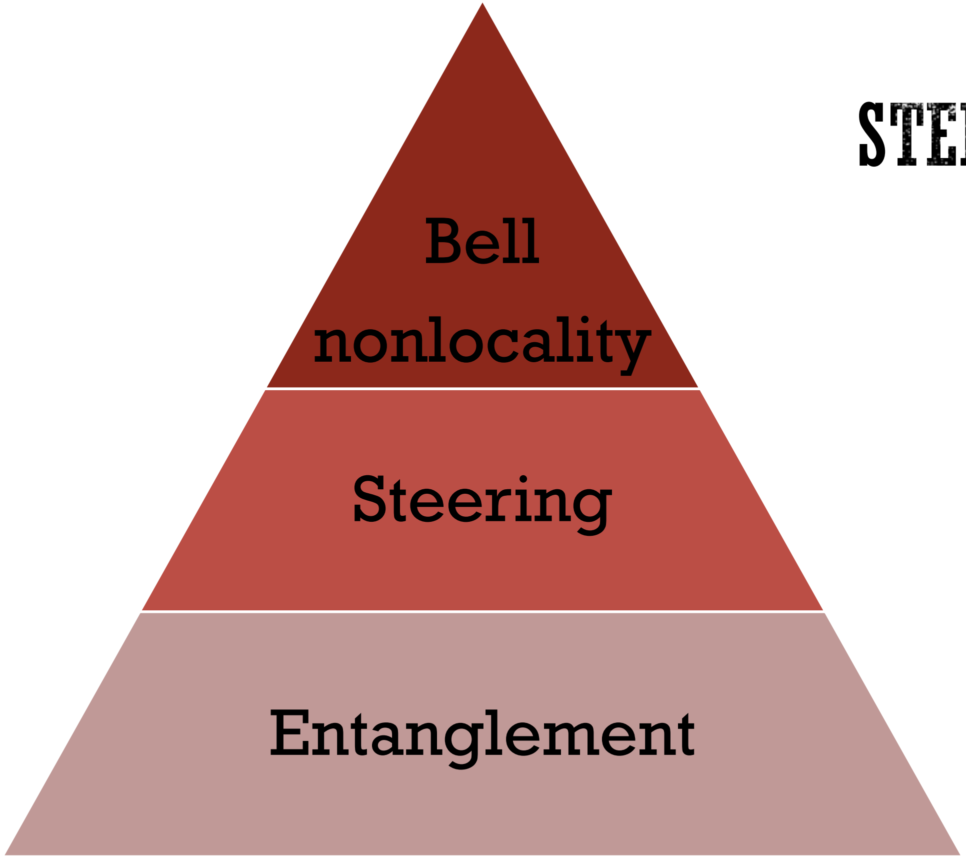

STEERING

Failure of Hybrid LHV – LQS model

$$p(a,b|x,y) = \int d\lambda p(\lambda) p(a|x,\lambda) \mathrm{Tr}[E_{b|y}] \sigma_\lambda^B$$

ENTANGLEMENT

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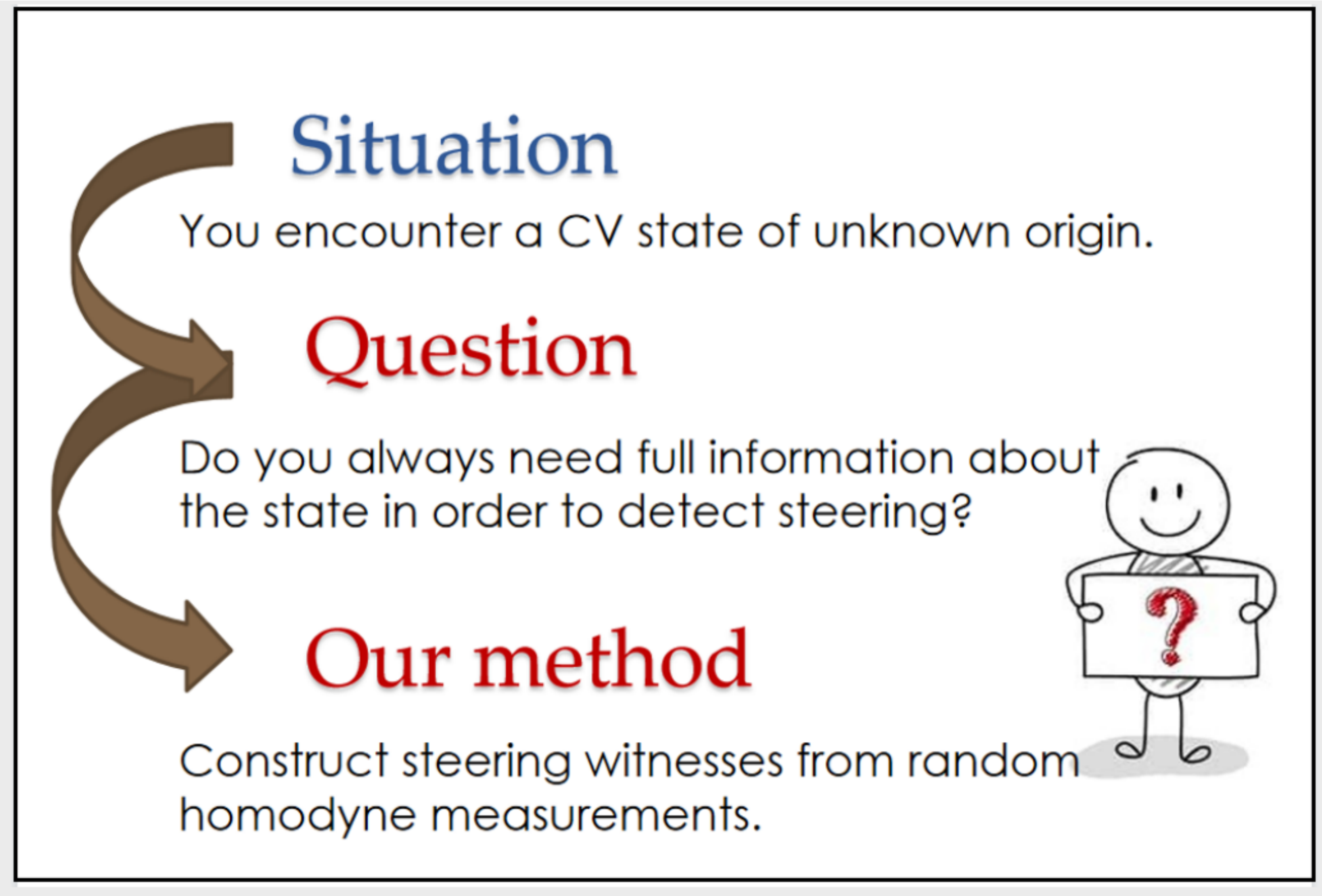
CONTENTS

- Continuous Variable States
  - Gaussian states
  - Covariance matrix (variances)
- Gaussian Steering
  - Witnesses method
- Detection strategy
  - Random measurements
  - Semidefinite programming



CONTINUOUS VARIABLE STATES

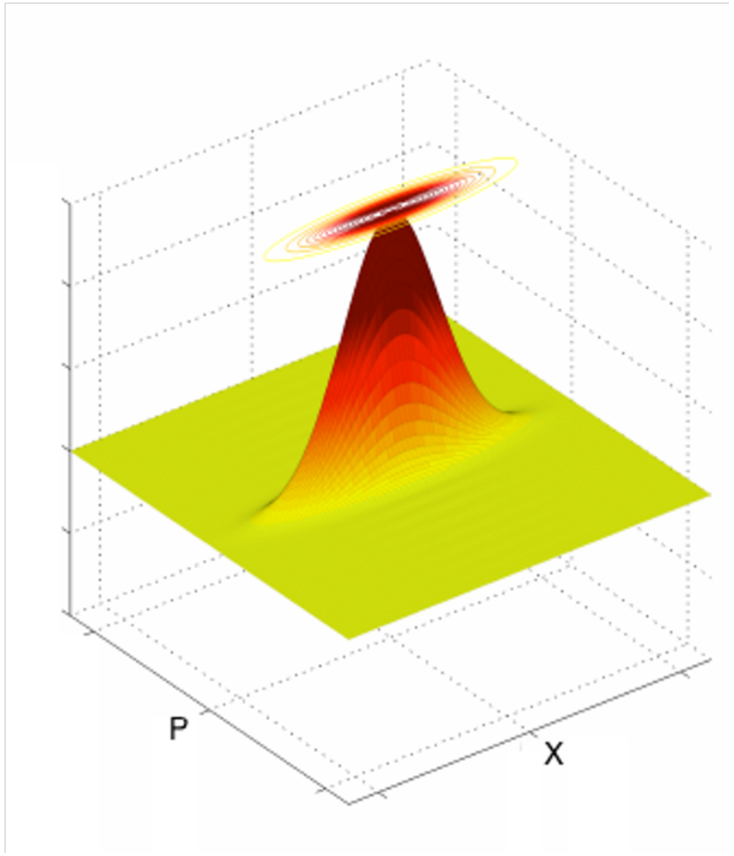
N bosonic modes: $\mathcal{H} = \bigotimes_{k=1}^{N} \mathcal{H}_k$

$$\hat{R}^{\mathrm{T}} \equiv (\hat{R}_1, \ldots, \hat{R}_{2N}) = (\hat{x}_1, \hat{p}_1, \ldots, \hat{x}_N, \hat{p}_N)$$

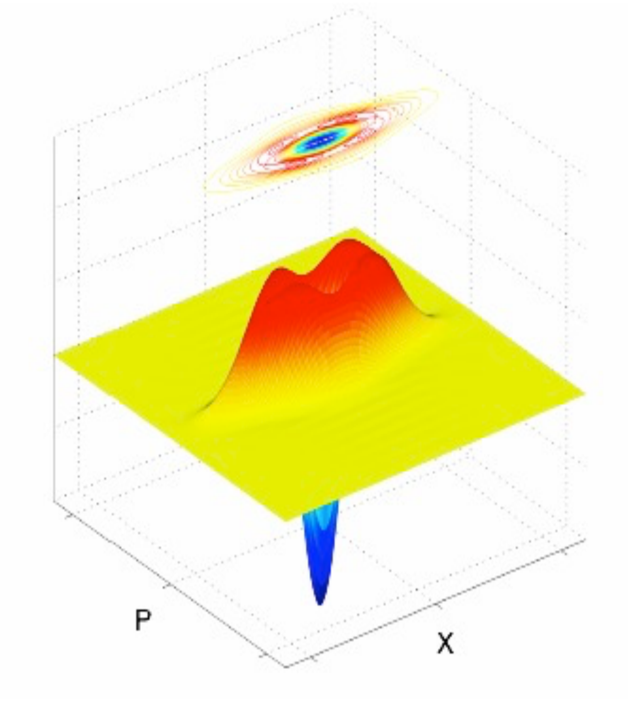
CCM: $[\hat{R}_i, \hat{R}_j] = \mathrm{i}\Omega_{ij}\hat{\mathrm{I}}, \quad i, j = 1, \ldots, 2N$

where $\Omega_{ij} \equiv [\Omega]_{ij}$ are the elements of:

$$\Omega_N = \bigoplus_1^N \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$



FOURIER-WEYL RELATION



- Complete set of operators: Weyl displacement operators

$$\hat{D}(r) = e^{ir^T \Omega_N \hat{R}}$$

$r^T = (x_1, p_1, \ldots, x_N, p_N)$ - a real vector of phase space variables.

- Fourier-Weyl relation

$$\hat{\rho} = \frac{1}{(2\pi)^{2N}} \int_{\mathbb{R}^{2N}} d^{2N} r \, \mathrm{Tr}[\hat{D}^\dagger(r)\hat{\rho}]\hat{D}(r),$$

where the characteristic function is given by

$$\chi_\rho = \mathrm{Tr}[\hat{D}^\dagger(r)\hat{\rho}]$$

GAUSSIAN STATES

- Characteristic function with zero first moments:

$$\chi_G = e^{\frac{1}{4} r^T \Omega_N^T \gamma_{AB} \Omega_N r},$$

- Covariance matrix (CM) γ_{AB}

$$\gamma_{ij} = \langle \{ \hat{R}_i - \langle \hat{R}_i \rangle, \hat{R}_j - \langle \hat{R}_j \rangle \}_+ \rangle_\rho .$$

- Uncertainty relation:

$$\gamma_{AB} + \mathrm{i}\Omega_N \geq 0.$$

- Bipartite CM

$$\gamma_{AB} = \begin{pmatrix} \gamma_A & \gamma_{12} \\ \gamma_{12}^{\mathrm{T}} & \gamma_B \end{pmatrix}$$

- Partial Gaussian measurements

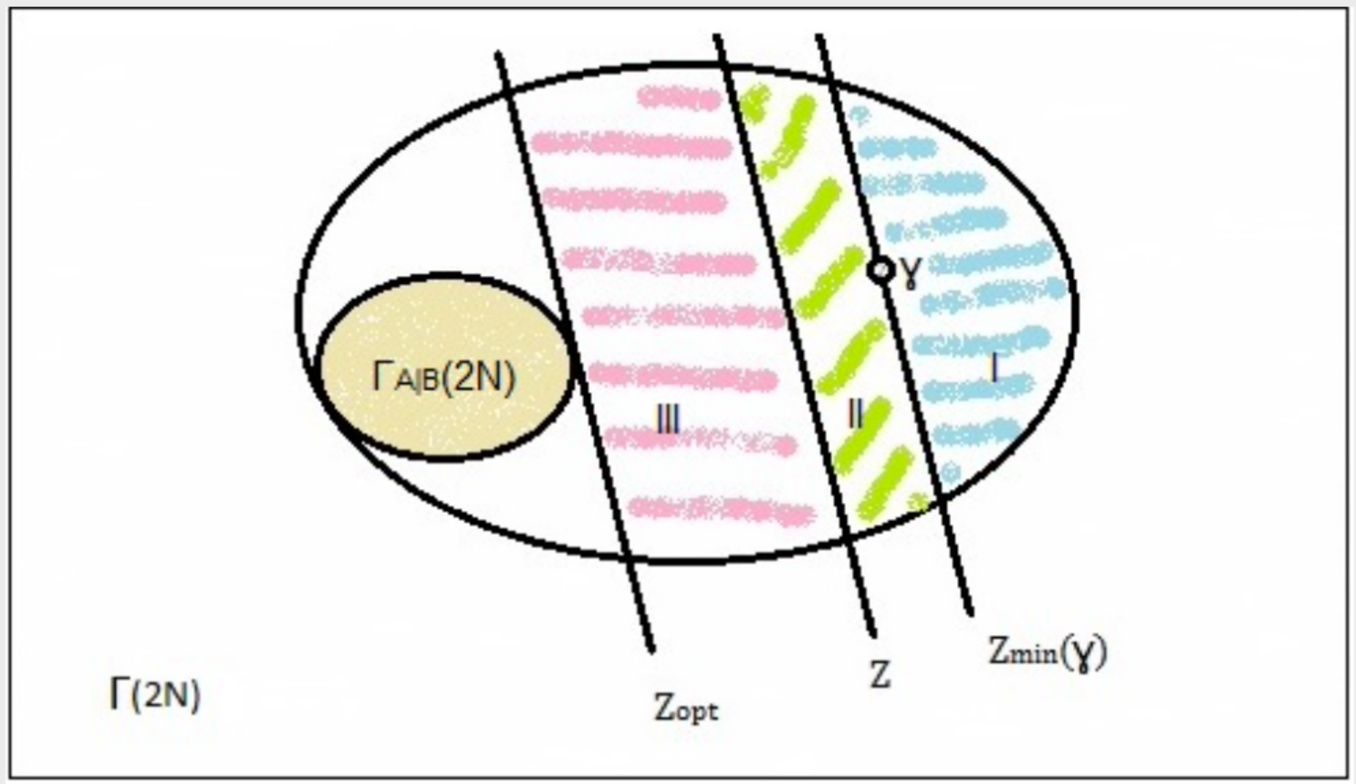
$$\mathrm{Tr}_A[(\hat{A} \otimes \hat{I}_B)\hat{\rho}],$$

- Gaussian operator $\hat{A}$ with CM T^A
- Conditional CM after measurement

$$\gamma_B^A = \gamma_B - \gamma_{12}^T (\gamma_A + T^A)^{-1} \gamma_{12}.$$

Theorem 2. *A bipartite quantum Gaussian state ρ_{AB} is Alice $\rightarrow$ Bob non-steerable by means of Gaussian measurements if and only if there exists a covariance matrix corresponding to Bob's system σ_B satisfying $\sigma_B + i\Omega_{N_B} \geq 0$, such that:*

$$\gamma_{AB} \geq 0_A \oplus \sigma_B.$$



T. Mihaescu, H. Kampermann, A. Isar, D. Bruss (to be published)

GAUSSIAN STEERING

The set of non-steerable CMs by Gaussian measurements, similarly to the set of all CMs, forms a closed convex subset of the space of all covariance matrices.

Hahn-Banach Theorem

This allows to completely describe the set of non-steerable CMs by a family of linear inequalities representing the steering witnesses (SWs).

Theorem 4. *A CM* γ_{AB} *of two parties consisting of* $N = N_A + N_B$ *modes is Alice to Bob steerable by means of Gaussian measurements if and only if there exists a* Z *such that:*

$$\mathrm{Tr}[Z\gamma_{AB}] < 1,$$

where Z *is a real symmetric* $2N \times 2N$ *matrix satisfying*

$$Z \geq 0 \quad \textit{and} \quad \mathrm{str}[Z_B] \geq \frac{1}{2},$$

where Z_B *denotes the principal submatrix of* Z *belonging to the subsystem of Bob. Matrices* Z *are called steering witnesses based on second moments.*

T. Mihaescu, H. Kampermann, A. Isar, D. Bruss (to be published)

STEERING WITNESSES

str – stands for the ”symplectic trace” which is the sum of singular eigenvalues of the matrix.

We are going to construct the witness from the measurements.

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$$Z_B + i\frac{1}{2N_B}\Omega_B \geq 0$$

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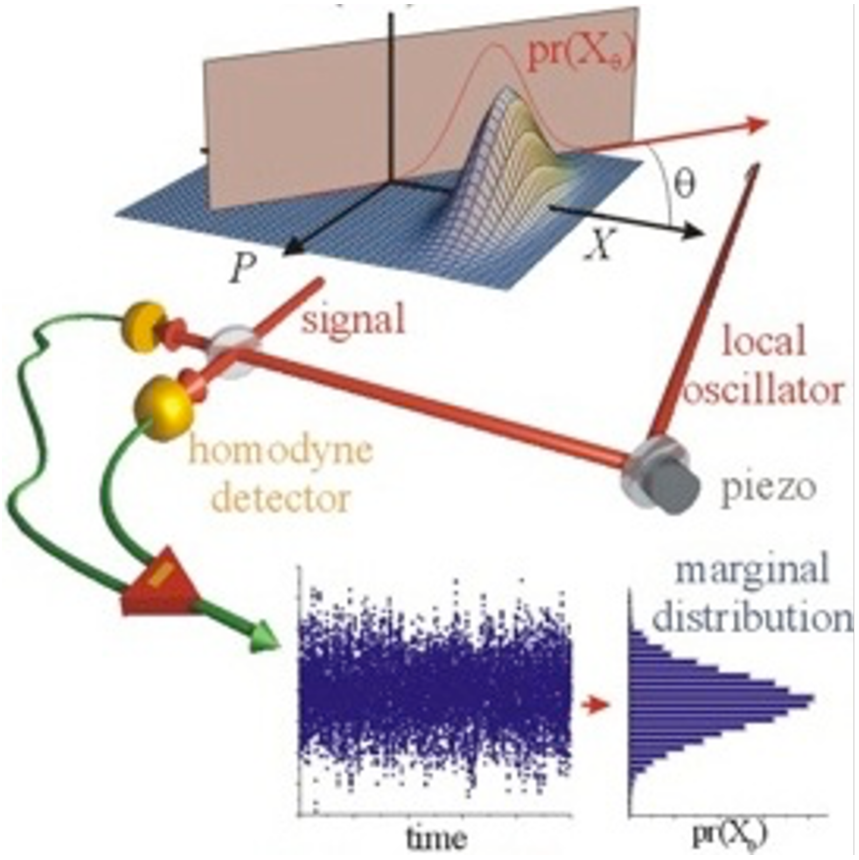
HOMODYNE DETECTION



- Measurement direction θ
- Quadrature to be measured

$$\hat{X}_\theta = \hat{x}\cos\theta + \hat{p}\sin\theta$$

- Marginal probability distribution $Pr(\hat{X}_\theta)$
- The variance

$$\sigma^2 = \langle \hat{X}_\theta^2 \rangle - \langle \hat{X}_\theta \rangle^2 = Tr[P\gamma]$$

where

$$P = \begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{pmatrix}$$

DETECTION SCHEME

- For two-mode states

$$\hat{k} = \exp(\mathrm{i}\varphi)\cos\phi\;\hat{a} + \sin\phi\;\hat{b}$$

- Generalized quadrature

$$\hat{x}_\theta = \frac{\exp(-\mathrm{i}\theta)\hat{k} + \exp(\mathrm{i}\theta)\hat{k}^\dagger}{\sqrt{2}}$$

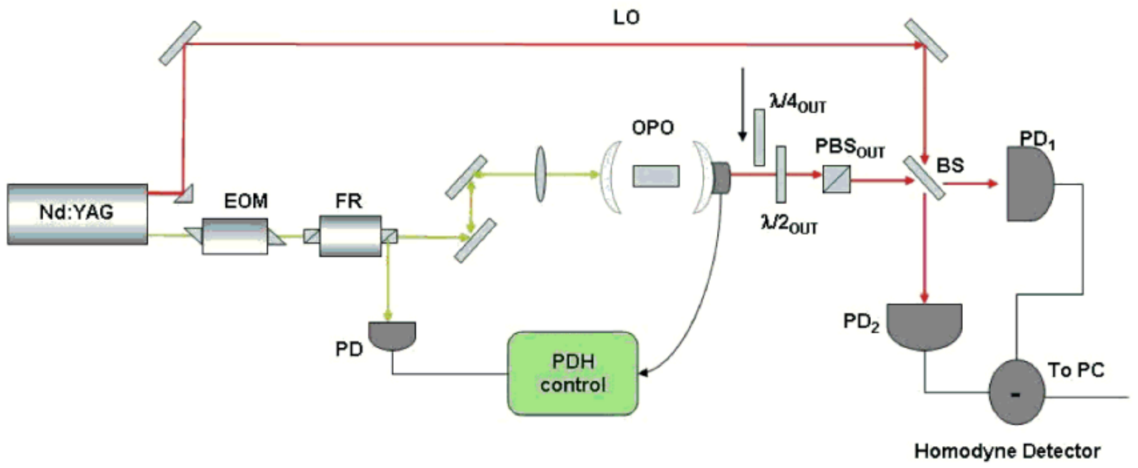


FIG. 1: Experimental setup: A type-II OPO containing a periodically poled crystal (PPKTP) is pumped by the second harmonic of a Nd:YAG laser. At the OPO output, a half-wave plate ($\lambda/2_{\mathrm{out}}$), a quarter-wave plate ($\lambda/4_{\mathrm{out}}$) and a $\mathrm{PBS}_{\mathrm{out}}$ select the mode for homodyning. The resulting electronic signal is acquired via a PC module.

A'Ariano et. al. *Full characterization of Gaussian bipartite entangled states by a single homodyne detector*

OPTIMIZATION ALGORITHM

$$\begin{aligned}
\text{minimize}_x: \quad & \mathbf{c}\cdot\mathbf{m} \\
\text{subject to}: \quad & Z = \sum_i c_i P_i \\
& Z \geq 0 \\
& Z_A + ix\Omega_1 \geq 0 \\
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where $\mathbf{m} = \mathrm{Tr}(\mathbf{P}\gamma)$, with $\mathbf{P}$ the vector of measurements P_i.

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Performe a random measurement

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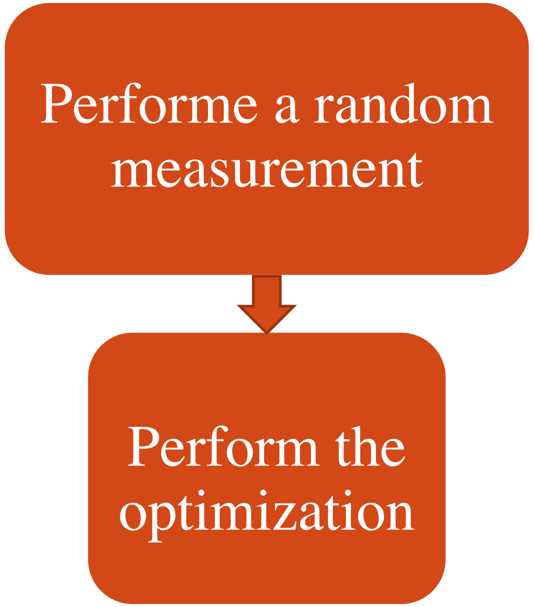

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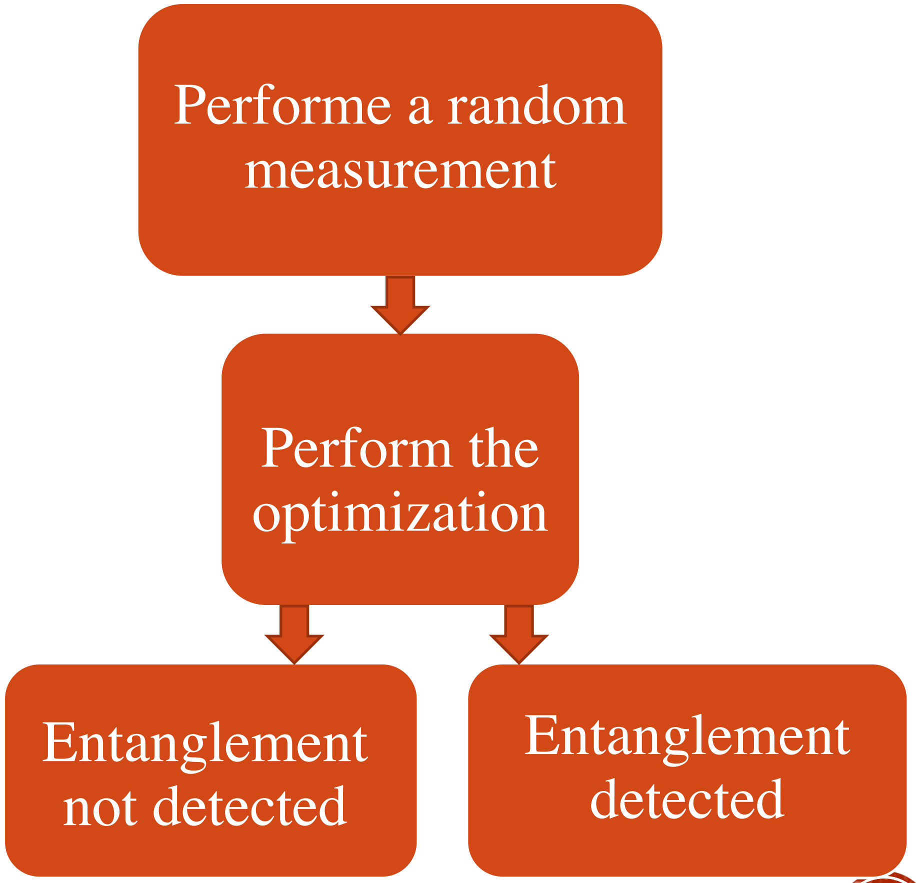

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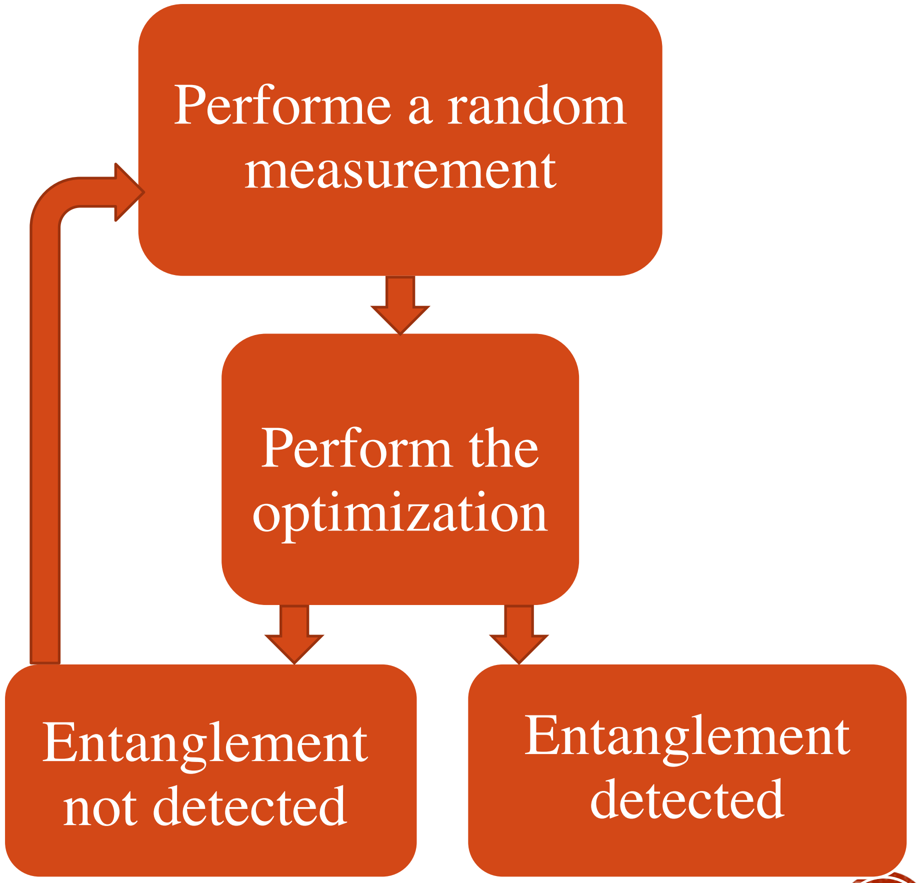

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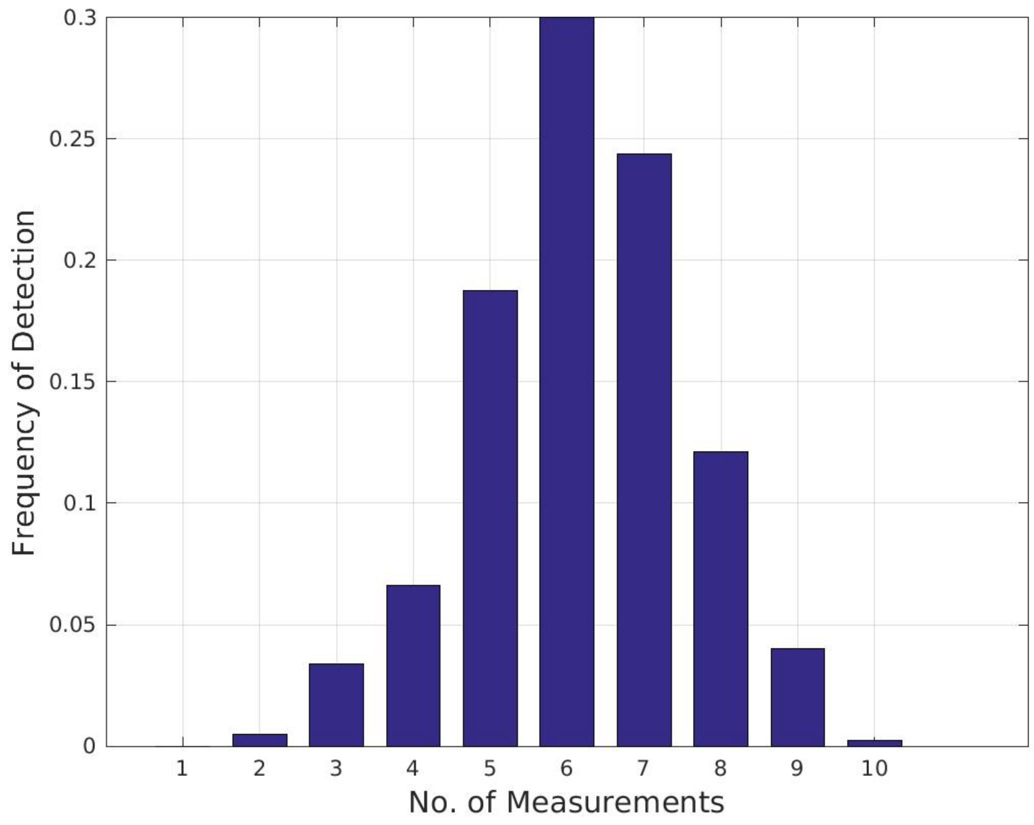
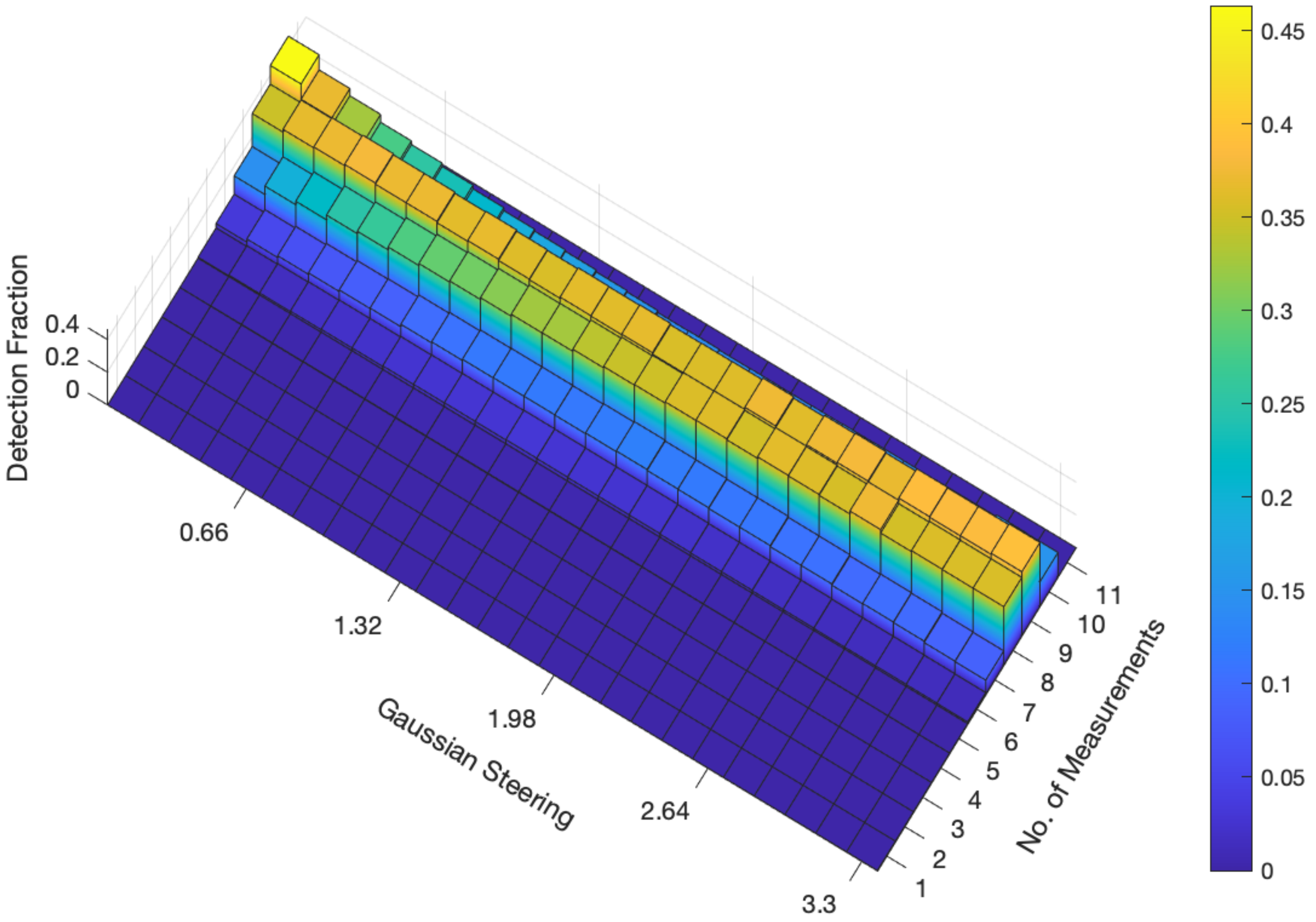
FIXED VALUE OF STEERING

- Two-mode covariance matrix
- Full tomography: 10 independent

Measurements

- The data is normalized such that it sums up to 1.





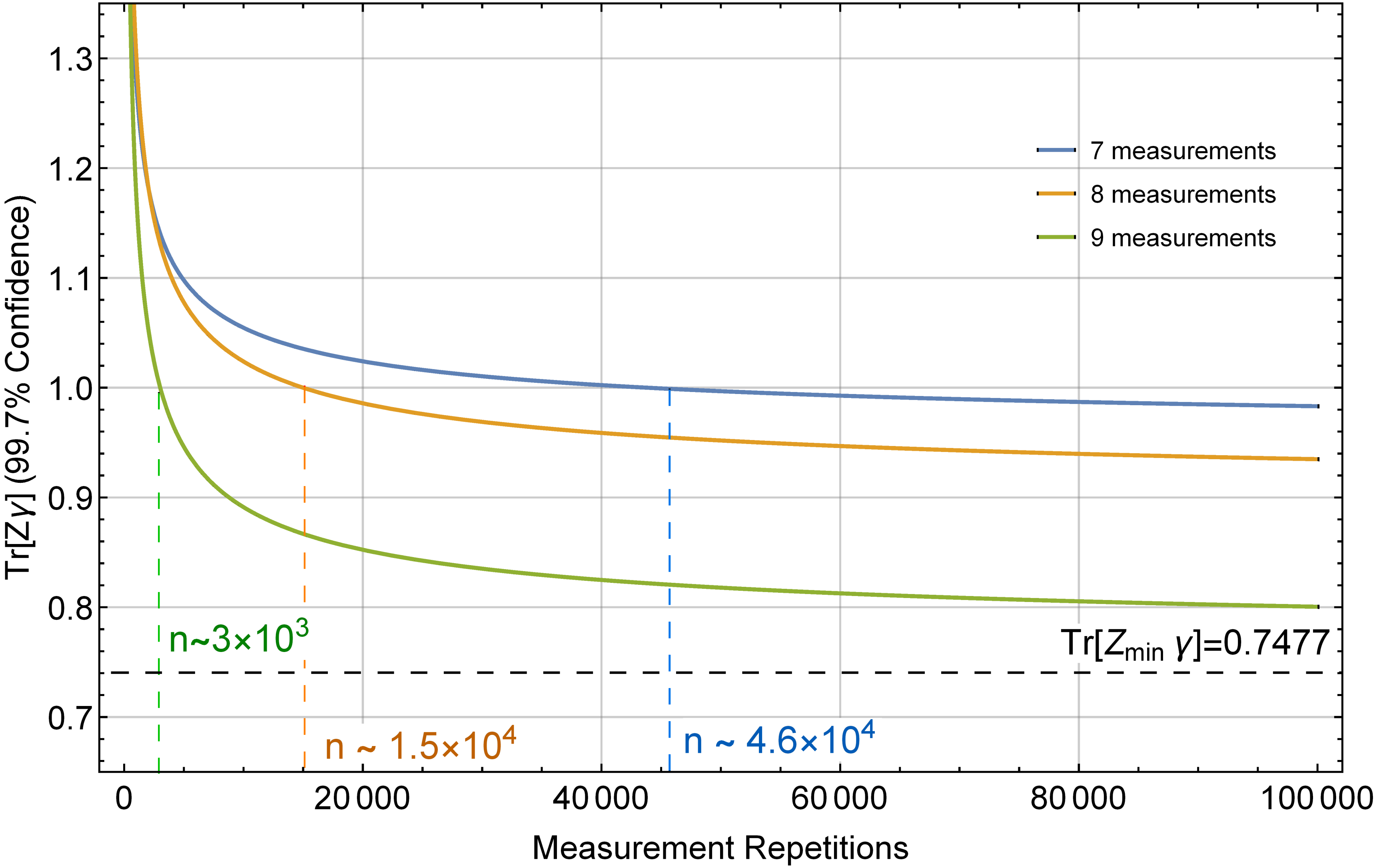
T. Mihaescu, H. Kampermann, A. Isar, D. Bruss (to be published)

DETECTION IN RANDOM COVARIANCE MATRICES

Fraction of steering detection for two-mode randomly generated CMs.

It represents 5 × 10^5 runs of the algorithm where measurement directions are added successively and SW is evaluated at every round until steering is certified.

The data is normalised for every value of steering such that they sum up to 1.



$$\Delta\bar{Z} = \sqrt{\frac{2}{n-1}}\sqrt{\sum_i c_i^2 m_i^2}.$$

T. Mihaescu, H. Kampermann, A. Isar, D. Bruss (to be published)

STATISTICAL ANALISYS

The statistical estimate of Z¯with the maximum of 3σ confidence interval.

The horizontal black dashed line indicates the minimal value of the witness for the considered CM Tr[Zminγ] = 0.7477.

The vertical dashed lines indicate the number of measurement repetitions required to detect Gaussian steering with 7 (blue), 8 (orange) and 9 (green) measurement settings.

THANK YOU FOR ATTENTION!