Messages of the deformed spacetime via quasinormal modes

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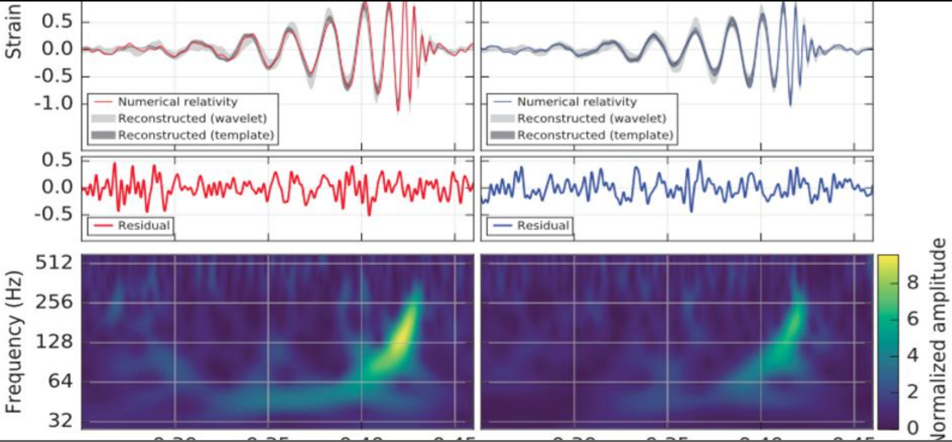

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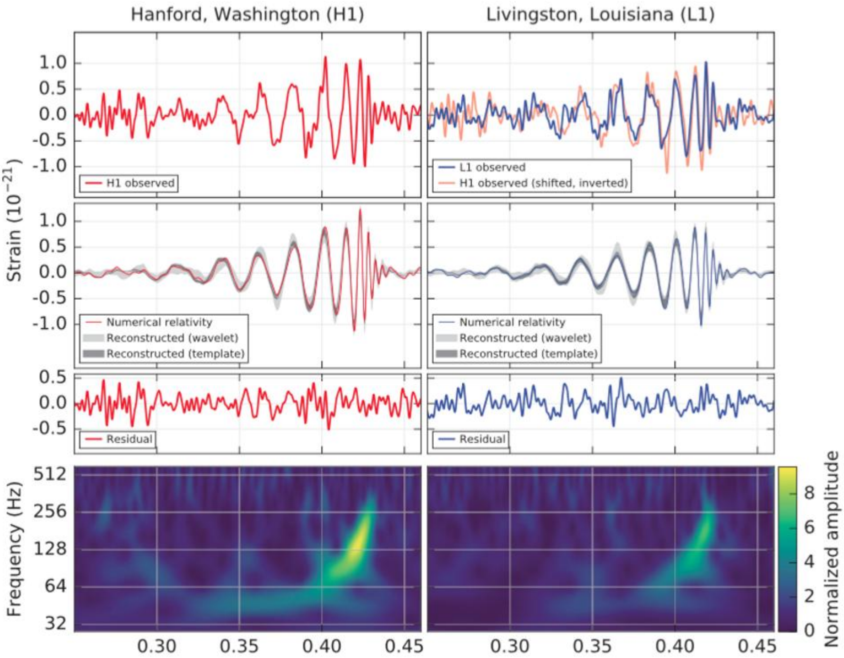
Detection of the gravitational waves can help better understanding of structure of space-time

Dominant stage of the perturbed BH are dumped oscillations of the geometry or matter fields (Quasinormal modes)



Hanford, Washington (H1)
Livingston, Louisiana (L1)
Strain (10^{-21})
H1 observed
L1 observed
H1 observed (shifted, inverted)
Numerical relativity
Reconstructed (wavelet)
Reconstructed (template)
Residual
Frequency (Hz)
Normalized amplitude
1.0
0.5
0.0
-0.5
-1.0
512
256
128
64
32
0.30
0.35
0.40
0.45
0
2
4
6
8

Noncommutative geometry

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Approaches to NC geometry $\star$-product, NC spectral triple, NC vierbein formalism, matrix models,...

NC space-time from the angular twist

Twist is used to deform a symmetry Hopf algebra
Twist $\mathcal{F}$ is invertible bidifferential operator from the universal enveloping algebra of the symmetry algebra

We work in 4D and deform the space-time by the following twist

$$\mathcal{F} = \mathrm{e}^{-\frac{i}{2}\theta_{ab}X^a \bigotimes X^b}$$

$[X^a, X^b] = 0, \quad$ a,b=1,2 $\qquad X_1 = \partial_0$ and $X_2 = x\partial_y - y\partial_x$

$$\mathcal{F} = \mathrm{e}^{\frac{-ia}{2}(\partial_0 \otimes (x\partial_y - y\partial_x) - (x\partial_y - y\partial_x)\otimes \partial_0)}$$

Bilinear maps are deformed by twist!
Bilinear map μ
$\mu : X \times Y \to Z$
$\mu_\star = \mu \mathcal{F}^{-1}$

Commutation relations between coordinates are:

$[\hat{x}^0, \hat{x}] = ia\hat{y},$ All other commutation relations are zero

$[\hat{x}^0, \hat{y}] = -ia\hat{x}$

Our approach: deform space-time by an Abelian twist to obtain commutation relations

Angular twist in curved coordinates $X_1 = \partial_0$ and $X_2 = \partial_\varphi$

-supose that metric tensor $g_{\mu\nu}$ does not depend on t and φ coordinates

-Hodge dual becomes same as in commutative case

Scalar $U(1)_\star$ gauge theory

If a one-form gauge field $\hat{A} = \hat{A}_\mu \star dx^\mu$ is introduced to the model through a minimal coupling, the relevant action becomes

$$
\begin{aligned}
S[\hat{\phi}, \hat{A}] &= \int \Big(d\hat{\phi} - i\hat{A} \star \hat{\phi} \Big)^+ \wedge_\star *_H \Big(d\hat{\phi} - i\hat{A} \star \hat{\phi} \Big) \\
&\quad - \int \frac{\mu^2}{4!} \hat{\phi}^+ \star \hat{\phi} \epsilon_{abcd}\; e^a \wedge_\star e^b \wedge_\star e^c \wedge_\star e^d \\
&= \int d^4x \sqrt{-g} \star \Big(g^{\mu\nu} \star D_\mu \hat{\phi}^+ \star D_\nu \hat{\phi} - \mu^2 \hat{\phi}^+ \star \hat{\phi} \Big)
\end{aligned}
$$

After expanding action and varying with respect to Φ^{+} EOM is

$$g^{\mu\nu}\left(D_\mu D_\nu\phi - \Gamma^{\lambda}_{\mu\nu}D_\lambda\phi\right) - \frac{1}{4}\theta^{\alpha\beta}g^{\mu\nu}\left(D_\mu(F_{\alpha\beta}D_\nu\phi) - \Gamma^{\lambda}_{\mu\nu}F_{\alpha\beta}D_\lambda\phi\right.$$

$$\left.-2D_\mu(F_{\alpha\nu}D_\beta\phi) + 2\Gamma^{\lambda}_{\mu\nu}F_{\alpha\lambda}D_\beta\phi - 2D_\beta(F_{\alpha\mu}D_\nu\phi)\right) = 0$$

Scalar field in the Reissner–Nordström background

RN metric tensor is

$$g_{\mu\nu} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & -\frac{1}{f} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin^2\theta \end{bmatrix}$$

with $f = 1 - \frac{2MG}{r} + \frac{Q^2 G}{r^2}$ which gives two horizons (r_+ and r_-)
Q-charge of RN BH
M-mass of RN BH
Non-zero components of gauge fields are $A_0 = -\frac{qQ}{r}$ i.e. $F_{r0} = \frac{qQ}{r^2}$
q-charge of scalar field

EOM for scalar field in RN space-time

$$\left(\frac{1}{f}\partial_t^2 - \Delta + (1-f)\partial_r^2 + \frac{2MG}{r^2}\partial_r + 2iqQ\frac{1}{rf}\partial_t - \frac{q^2Q^2}{r^2 f}\right)\phi$$
$$+\frac{aqQ}{r^3}\Big(\big(\frac{MG}{r} - \frac{GQ^2}{r^2}\big)\partial_\varphi + rf\partial_r\partial_\varphi\Big)\phi = 0$$

where a is $\theta^{t\varphi}$

Assuming ansatz $\phi_{lm}(t,r,\theta,\varphi) = R_{lm}(r)e^{-i\omega t}Y_l^m(\theta,\varphi)$ we got equation for radial part

$$fR''_{lm} + \frac{2}{r}(1-\frac{MG}{r})R'_{lm} - \Big(\frac{l(l+1)}{r^2} - \frac{1}{f}(\omega - \frac{qQ}{r})^2\Big)R_{lm}$$
$$-ima\frac{qQ}{r^3}\Big(\big(\frac{MG}{r} - \frac{GQ^2}{r^2}\big)R_{lm} + rfR'_{lm}\Big) = 0 \tag{1}$$

NC QNM solutions

QNM

-special solution of equation

-damped oscillations of a perturbed black hole

A set of the boudary condition which leads to this solution is the following: at the horizon, the QNMs are purely incoming, while in the infinity the QNMs are purely outgoing

Continued fraction method

To get form

$$\frac{d^2\psi}{dy^2} + V\psi = 0$$

y must be

$$y = r + \frac{r_+}{r_+ - r_-}\Big(r_+ - iamqQ\Big)\ln(r - r_+) - \frac{r_-}{r_+ - r_-}\Big(r_- - iamqQ\Big)\ln(r - r_-)$$

y is modified Tortoise RN coordinate

Asymptotic form of the eq. (1)

$$R(r) \to \begin{cases} Z^{out} e^{i\Omega y} y^{-1-i\frac{\omega qQ-\mu^2 M}{\Omega} - amqQ\Omega} & \text{za } y \to \infty \\ Z^{in} e^{-i\left(\omega - \frac{qQ}{r_+}\right)\left(1+iam\frac{qQ}{r_+}\right)y} & \text{za } y \to -\infty \end{cases}$$

Combining assymptotic forms, we get general solution in the form

$$R(r) = e^{i\Omega r}(r - r_-)^{\epsilon} \sum_{n=0}^{\infty} a_n \left(\frac{r - r_+}{r - r_-}\right)^{n+\delta} \tag{2}$$

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$$\delta = -i\frac{r_+^2}{r_+ - r_-}\left(\omega - \frac{qQ}{r_+}\right), \qquad \epsilon = -1 - iqQ\frac{\omega}{\Omega} + i\frac{r_+ + r_-}{2\Omega}\left(\Omega^2 + \omega^2\right),$$

$$\Omega = \sqrt{\omega^2 - \mu^2}$$

Putting general form (2) to eq (1) we get 6-term recurrence relations for a_n:

$$
\begin{aligned}
A_n a_{n+1} + B_n a_n + C_n a_{n-1} + D_n a_{n-2} + E_n a_{n-3} + F_n a_{n-4} = 0, \\
A_3 a_4 + B_3 a_3 + C_3 a_2 + D_3 a_1 + E_3 a_0 = 0, \\
A_2 a_3 + B_2 a_2 + C_2 a_1 + D_2 a_0 = 0, \\
A_1 a_2 + B_1 a_1 + C_1 a_0 = 0, \\
A_0 a_1 + B_0 a_0 = 0,
\end{aligned}
$$

$$
\begin{aligned}
A_n &= r_+^{\mathbf{3}}\alpha_n,\\
B_n &= r_+^{\mathbf{3}}\beta_n - iamqQ(r_+ - r_-)r_+(n+\delta) - \frac{1}{2}iamqQ(r_+ + r_-)r_+\\
&\quad + iamqQr_+r_- - 3r_+^{\mathbf{2}}r_-\alpha_{n-\mathbf{1}},\\
C_n &= r_+^{\mathbf{3}}\gamma_n + 3r_+r_-^{\mathbf{2}}\alpha_{n-\mathbf{2}} + iamqQ(r_+ - r_-)(2r_+ + r_-)(n+\delta-1)\\
&\quad - iamqQ(r_+ - r_-)r_+\epsilon \qquad + \frac{1}{2}iamqQ(r_+ + r_-)(2r_+ + r_-)\\
&\quad - 3iamqQr_+r_- + amqQ\Omega(r_+ - r_-)^{\mathbf{2}}r_+ - 3r_+^{\mathbf{2}}r_-\beta_{n-\mathbf{1}}+,\\
D_n &= -r_-^{\mathbf{3}}\alpha_{n-\mathbf{3}} + 3r_+r_-^{\mathbf{2}}\beta_{n-\mathbf{2}} - 3r_+^{\mathbf{2}}r_-\gamma_{n-\mathbf{1}} + iamqQ(r_+^{\mathbf{2}} - r_-^{\mathbf{2}})\epsilon + 3iamqQr_+r_-\\
&\quad - amqQ\Omega(r_+ - r_-)^{\mathbf{2}}r_- - iamqQ(r_+ - r_-)(r_+ + 2r_-)(n+\delta-2)\\
&\quad - \frac{1}{2}iamqQ(r_+ + r_-)(r_+ + 2r_-),\\
E_n &= 3r_+r_-^{\mathbf{2}}\gamma_{n-\mathbf{2}} - r_-^{\mathbf{3}}\beta_{n-\mathbf{3}} + iamqQ(r_+ - r_-)r_-(n+\delta-3)\\
&\quad - iamqQ(r_+ - r_-)r_-\epsilon + \frac{1}{2}iamqQ(r_+ + r_-)r_-\,iamqQr_+r_-,\\
F_n &= -r_-^{\mathbf{3}}\gamma_{n-\mathbf{3}},
\end{aligned}
$$

$$\alpha_n = (n+1)\Big[n+1-2i\frac{r_+}{r_+-r_-}(\omega r_+ - qQ)\Big],$$

$$\beta_n = \epsilon + (n+\delta)(2\epsilon - 2n - 2\delta) + 2i\Omega(n+\delta)(r_+ - r_-) - l(l+1) - \mu^2 r_-^2$$
$$+ \frac{2\omega r_-^2}{r_+ - r_-}(\omega r_+ - qQ) - \frac{2r_-^2}{(r_+ - r_-)^2}(\omega r_+ - qQ)^2 + 4\omega r_-(\omega r_+ - qQ)$$
$$- \frac{2r_-}{r_+ - r_-}(\omega r_+ - qQ)^2 + (r_+ - r_-)\Big[i\Omega + 2\omega(\omega r_+ - qQ) - \mu^2(r_+ + r_-)\Big],$$

$$\gamma_n = \epsilon^2 + (n+\delta-1)(n+\delta-1-2\epsilon) + \Big(\omega r_- - \frac{r_-}{r_+ - r_-}(\omega r_+ - qQ)\Big)^2$$

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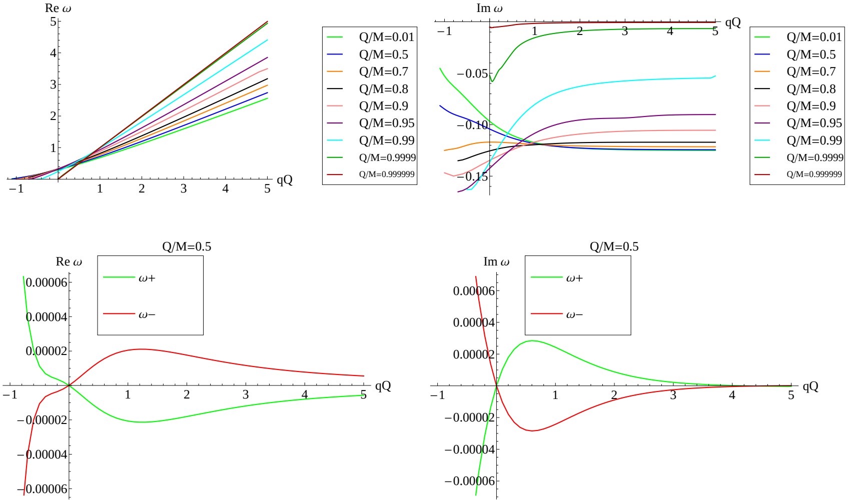
- Gauss elimination procedure allows to reduce $n+1$-recurrence relation to n-recurrence relation
- 3-term relation

$$\alpha_n a_{n+1} + \beta_n a_n + \gamma_n a_{n-1} = 0, \qquad \alpha_0 a_1 + \beta_0 a_0 = 0$$

gives following equation

$$0 = \beta_0 - \cfrac{\alpha_0 \gamma_1}{\beta_1 - \cfrac{\alpha_1 \gamma_2}{\beta_2 - \cfrac{\alpha_2 \gamma_3}{\beta_3 - \cdots \cfrac{\alpha_n \gamma_{n+1}}{\beta_{n+1} - \cdots}}}}$$


Re ω
qQ
Q/M=0.01
Q/M=0.5
Q/M=0.7
Q/M=0.8
Q/M=0.9
Q/M=0.95
Q/M=0.99
Q/M=0.9999
Q/M=0.999999
Im ω
qQ
Q/M=0.5
Re ω
$\omega+$
$\omega-$
qQ
Q/M=0.5
Im ω
$\omega+$
$\omega-$
qQ

Duality picture

We have another way to get the equation of motion for scalar field 1: Using the effective metric in commutative space

$$g_{\mu\nu} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & -\frac{1}{f} & 0 & -\frac{aqQ}{2}\sin^2\theta \\ 0 & 0 & -r^2 & 0 \\ 0 & -\frac{aqQ}{2}\sin^2\theta & 0 & -r^2\sin^2\theta \end{pmatrix} \tag{2}$$

Fermions

To check the duality picture, we have done the same procedure for fermions in RN metric (coupled to external EM field).

- Noncommutative space with pure RN metric
- Commutative space with modified RN metric

The result is the same in both cases

$$i\gamma^{\mu}\Big(\partial_{\mu}\Psi - i\omega_{\mu}\Psi - iA_{\mu}\Psi\Big) - m\Psi - \frac{ia}{2}\frac{qQ}{r^{2}}\sqrt{f}\gamma^{1}\partial_{\phi}\Psi = 0.$$

Outlook

- We constructed Angular twist which induces angular noncommutativity
- Angular NC scalar and vector gauge theory is constructed
- EOM is solved with QNM boundary conditions for scalar field coupled to RN geometry
- But this is toy model!
- Plan for future is to calculate fermionic and gravitational QNMs and to compare it with results from LIGO, VIRGO, LISA...
- We want to understand physics of the effective metric