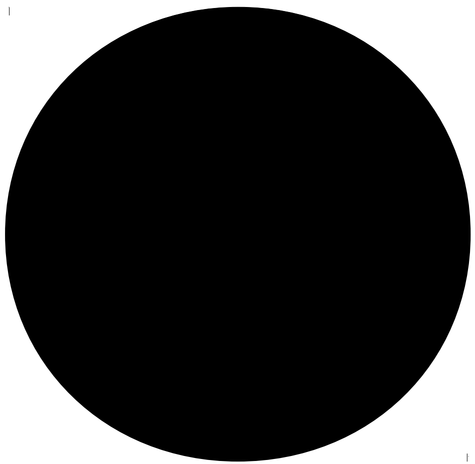
Towards non-AdS holography via solvable irrelevant deformations

Monica Guica

IphT, CEA Saclay

Introduction

- black hole entropy



$$\boxed{S_{BH} = \frac{\mathcal{A}_H}{4G}}$$

universal

Introduction

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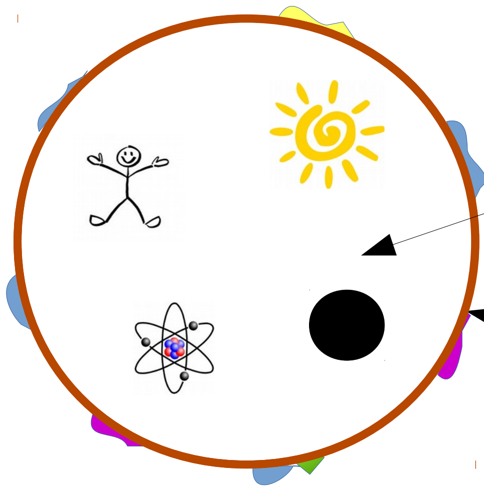


$$\boxed{S_{BH} = \frac{\mathcal{A}_H}{4G}}$$

universal

Holography

Gravitational theory
d+1 dimensions

⇵

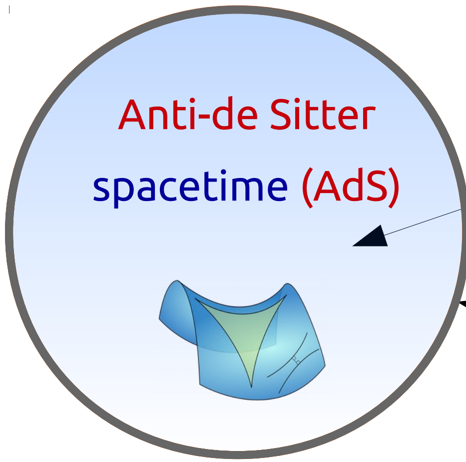
Field theory (no gravity)
d dimensions

✓ usual Quantum Mechanics

Introduction

- The AdS/CFT correspondence

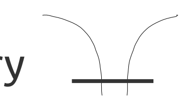
Holography



Gravitational theory on AdS_{d+1}

⇅

Conformal field theory CFT_d

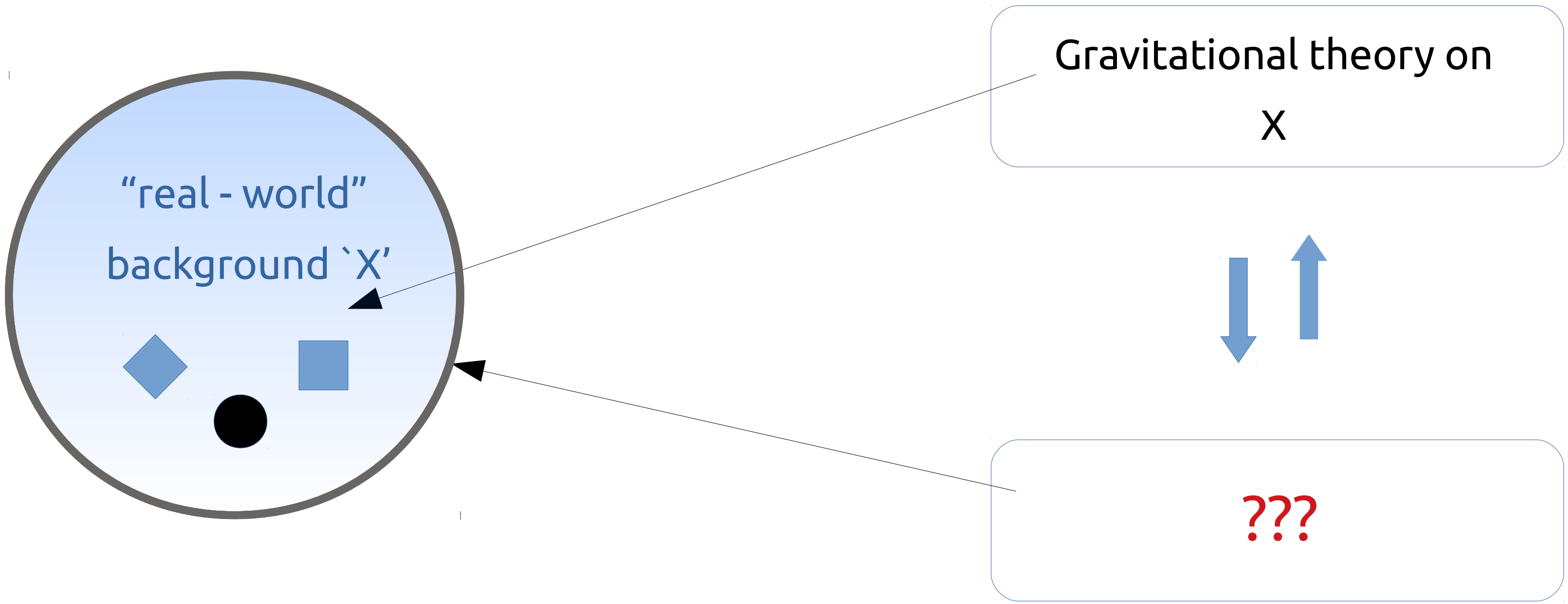
- ``top-down'' : concrete constructions in string theory  ← specific examples
- ``bottom-up'' : $\forall$ CFT_d with large N/large gap → gravity in AdS_{d+1} ← match symmetries, correlation f.

axiomatic properties of CFTs (primary operators & prop.) are **essential**

Introduction

- Quantum gravity in the real world ?

Holography

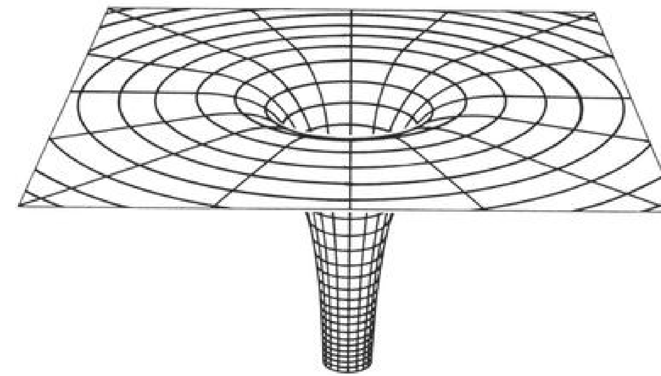


even most basic axiomatic properties of dual field theory are not known → **non-local**

no concrete examples (string theory)

Black holes to the rescue

- extreme Kerr black hole $GM^2 \simeq J$ e.g. GRS 1915+105



Near Horizon Extreme Kerr

$wAdS_3$

- the Kerr/CFT correspondence

gravity analysis
- infinite # of symmetries → Virasoro $\frac{1}{2}$ CFT$_2$
- entropy match $S_{BH} = \frac{A_H}{4G}$ ✔

M.G., Hartman, Song, Strominger '08

- universality (all extremal black holes have Virasoro + entropy match)

many people

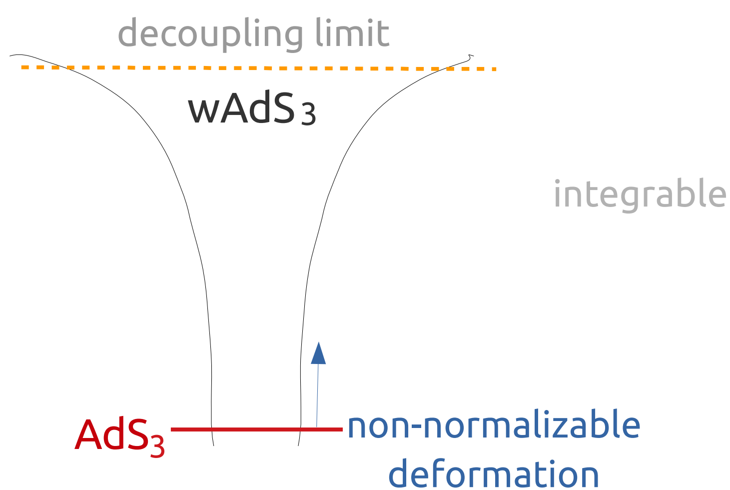
- scattering amplitudes match CFT$_2$ correlation functions ✔ (! but with $h \to h(p)$)

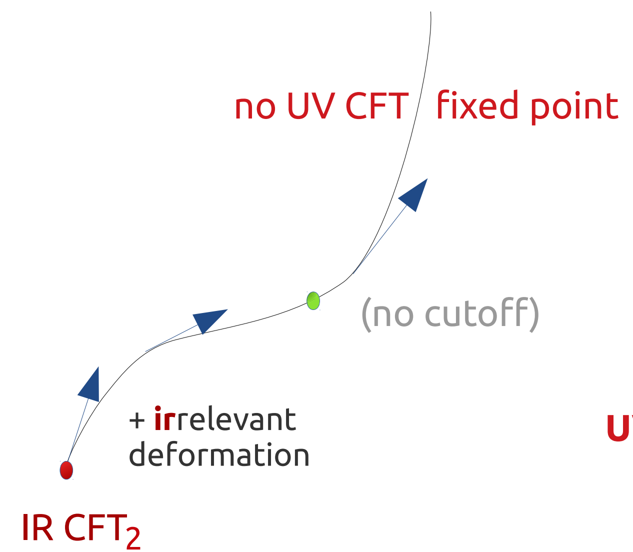
Bredberg, Hartman, Song, Strominger '09

- looks very similar to AdS_3/CFT_2, but different from it ← non-local

The "gist" of the correspondence

- universality → embed Kerr/CFT in string theory → control





UV complete

non-local

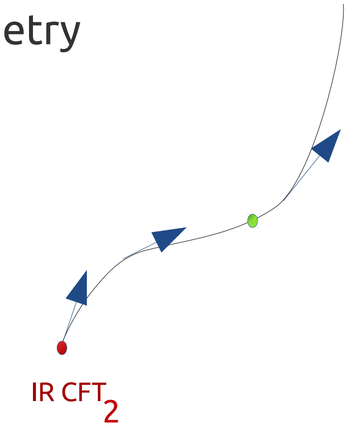
- irrelevant flow **hard** & very **unnatural** from RG perspective

- infer field theory properties from gravity:
  - ASG : Virasoro x Virasoro
  - scattering: CFT_2 with $h(p)$

"non-local CFT"

The question

non-local CFT$_2$ s $\equiv$ UV-complete non-local 2d QFTs with Virasoro x Virasoro symmetry



Do non-local CFT$_2$ s exist?

Yes ✓ No ✗

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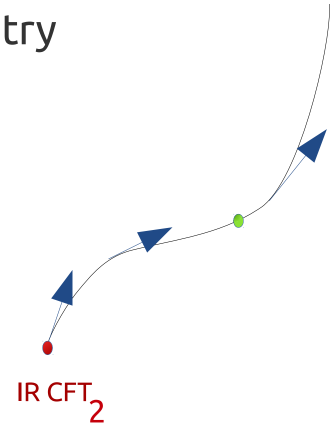
non-local CFT_2s $\equiv$ UV-complete non-local 2d QFTs with Virasoro x Virasoro symmetry



Do non-local CFT_2 s exist?

Yes ✓ No ✗

- how does one reconcile the Virasoro symmetry with the non-locality?
- consequences on the observables, such as correlation functions?
- axiomatic definition, classification …..
- …..
- non-AdS holography

The claims

non-local $\mathrm{CFT_2}$s $\equiv$ UV-complete non-local 2d QFTs with Virasoro x Virasoro symmetry

$T\bar{T}$ and $J\bar{T}$ -deformed CFTs are non-local CFTs

✓ well-defined via the Smirnov-Zamolodchikov construction

✓ infinite # of symmetries: Virasoro-KM x Virasoro-KM algebra

classically → field-dependent coordinate transformations

(pseudoconformal)

✓ @ least in $J\bar{T}$ – deformed CFTs → notion of a "primary" operator, whose correlation

functions are entirely fixed in terms of those of the undeformed CFT $h \to h(p)$

2 & 3 p.f.

Smirnov-Zamolodchikov deformations

- irrelevant deformations of 2d QFTs → bilinears of two (higher spin) conserved currents $J^A, \; J^B$

- define $\mathcal{O}_{J^A J^B}$:

$$\boxed{\lim_{y \to x} \epsilon^{\alpha\beta} J^A_\alpha(x) J^B_\beta(y) = \mathcal{O}_{J^A J^B} +} \text{ derivative terms}$$

Zamolodchikov '04

SZ '16

nice factorization properties

- deformation :

$$\boxed{\frac{\partial S(\mu)}{\partial \mu} = \int d^2x \, \mathcal{O}_{J^A J^B}(\mu)}$$

- examples:

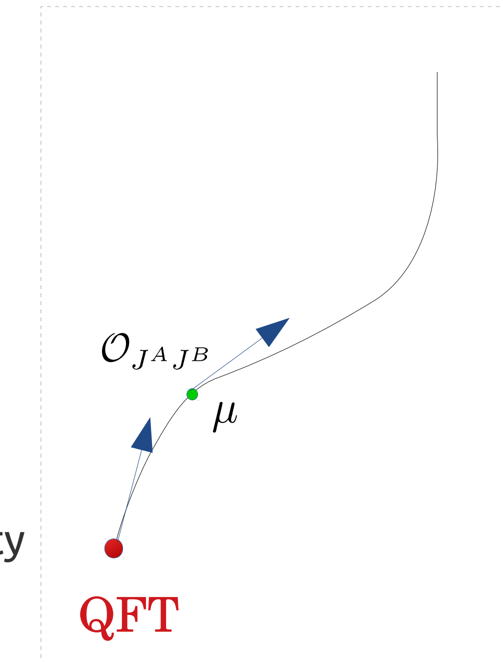
  universal

$$T\bar{T}: \quad J^A_\alpha = T_\alpha{}^A \, , \quad J^B_\beta = T_\beta{}^B \quad (\times \epsilon_{AB}) \qquad (2,2)$$

$$J\bar{T}: \quad J^A_\alpha = J_\alpha \, , \quad J^B_\beta = T_{\beta\bar{z}} \qquad (1,2)$$

~~Lorentz~~

$$\underbrace{SL(2,\mathbb{R})_L}_{\text{local \& conformal}} \times \underbrace{U(1)_R}_{\text{non-local!}}$$



- highly tractable : exact finite -size spectrum, S-matrix, preserves integrability

- deformed theory non-local (scale $\mu^{\#}$) but argued UV complete

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simpler than $T\bar{T}$! ← local & conformal

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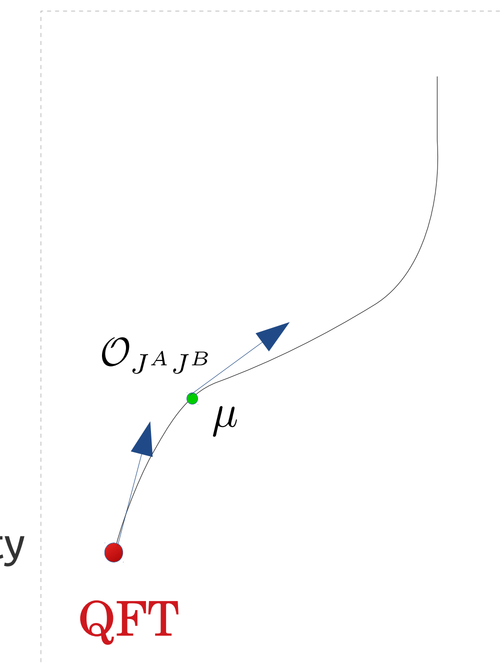

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$J\bar{T}:\quad J^A_\alpha = J_\alpha\,,\quad J^B_\beta = T_{\beta\bar{z}}$ ~~Lorentz~~ (1,2)

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same as extremal black holes ←

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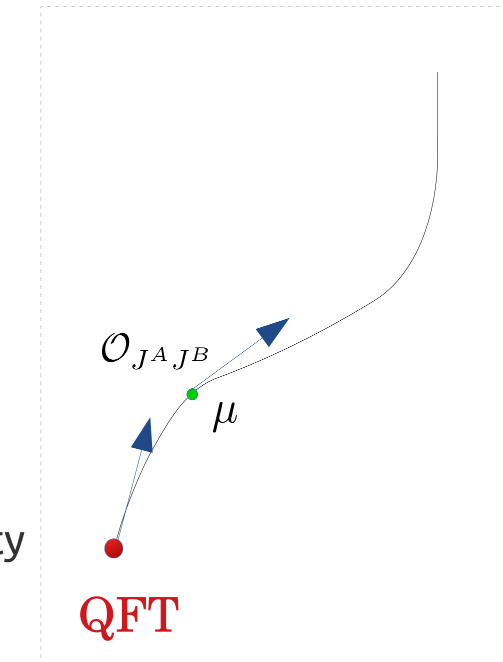

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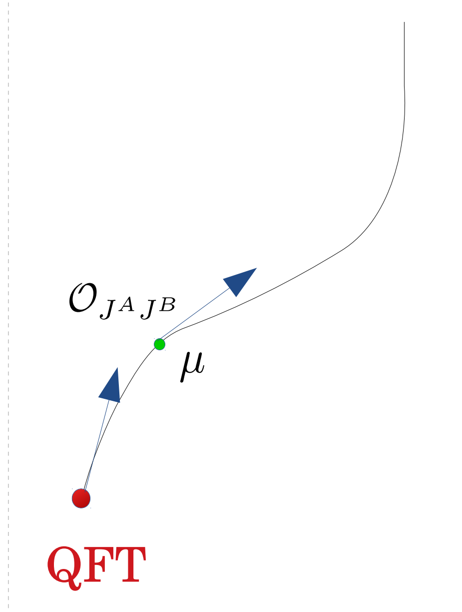
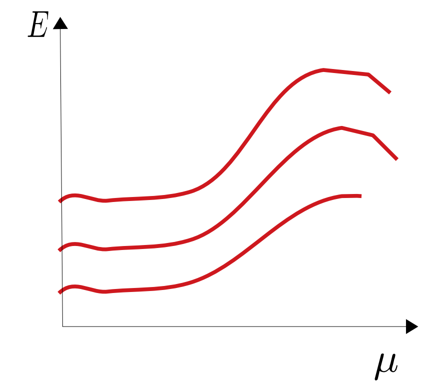


Flow of the energies and eigenstates

- place SZ-deformed theory on a cylinder (R) → Hilbert space unchanged, only $H(\mu)$ and its eigenvalues

$$\partial_\mu E_n = \langle n_\mu | \partial_\mu H | n_\mu \rangle \qquad \partial_\mu | n_\mu \rangle = \sum_{m \neq n} \frac{\langle m_\mu | \partial_\mu H | n_\mu \rangle}{E_n^\mu - E_m^\mu} | m_\mu \rangle \equiv \mathcal{X}_{J^A J^B} | n_\mu \rangle$$



- universal deformed energy spectrum determined only by the initial one

e.g. $T\bar{T}$ on seed CFT

$$E_\mu(R) = \frac{R}{2\mu}\left(\sqrt{1 + \frac{4\mu E_0}{R} + \frac{4\mu^2 P^2}{R^2}} - 1\right)$$

Hagedorn at high energy

- similar exact results for $J\bar{T}$ (λ): $SL(2,\mathbb{R})$ dimensions & U(1) charge

$$h(\lambda) = \tilde{h} + \lambda \tilde{q} \bar{p} + \frac{k\lambda^2}{4}\bar{p}^2 \qquad q(\lambda) = q + \frac{k\lambda}{2}\bar{p}$$

→ spectral flow with momentum-dependent parameter

Symmetries of $T\bar{T}$ and $J\bar{T}$ - deformed CFTs

Virasoro symmetry: abstract proof

MG'21

- flow of eigenstates on the cylinder

$$\partial_\mu |n_\mu\rangle = \mathcal{X}_{J^A J^B} |n_\mu\rangle \qquad \mathcal{X}_{J^A J^B} : \text{ QM well-defined}$$

- define $\tilde{L}^\mu_m$ via

$$\boxed{\partial_\mu \tilde{L}^\mu_m = [\mathcal{X}_{J^A J^B}, \tilde{L}^\mu_m]} \qquad \tilde{L}^\mu_m(\mu = 0) = L^{CFT}_m$$

and same for all other symmetry generators ($\tilde{\bar{L}}^\mu_m$, $\tilde{J}^\mu_m \ldots$)

→ well-defined quantum-mechanically, unambiguous

→ satisfy Virasoro algebra by construction, same **c** as undeformed CFT

→ $\tilde{L}^\mu_0 |n_\mu\rangle = \tilde{h} |n_\mu\rangle$, where $\tilde{h}$ are the undeformed conformal dimensions

- Virasoro algebra $\not\Rightarrow$ Virasoro symmetry → need conservation!

Virasoro symmetry: abstract proof

- conservation condition $$\left(\frac{\partial \tilde{L}^{\mu}_{m,S}(t)}{\partial t}\right)_H + \frac{i}{\hbar}[H, \tilde{L}^{\mu}_{m,H}] = 0$$

- universal $T\bar{T}$ – deformed spectrum holding for all eigenstates → $H = f(\tilde{L}^{\mu}_0, \bar{\tilde{L}}^{\mu}_0)$

$$E_\mu = \frac{1}{2\mu}\left(\sqrt{1 + 4\mu(\tilde{h} + \bar{\tilde{h}}) + 4\mu^2(\tilde{h} - \bar{\tilde{h}})^2} - 1\right)$$

$$P = \tilde{h} - \bar{\tilde{h}}$$

LeFloch, Mezei '19

- $\Rightarrow \quad [\tilde{L}^{\mu}_m, H] = \alpha_m(H,P)\, \tilde{L}^{\mu}_m$ $\qquad \alpha_m = \frac{1}{2\mu}\left[\sqrt{(1+2\mu H)^2 + 4\mu m\hbar(1+2\mu P) + 4\mu^2 m^2 \hbar^2} - (1+2\mu H)\right]$

- the following operators are then conserved

$$\tilde{L}^{\lambda}_{m,S}(t) \equiv e^{i\alpha_m(H,P)t}\tilde{L}_{m,S}(0)$$ → **Virasoro symmetries**

- only difference with standard CFT is that $\tilde{L}^{\mu}_0 \not\propto H$ & → α_m is operator-valued ($\alpha_m = m\hbar$ in CFT)

- similar results hold for $J\bar{T}$

Classical realization of the symmetries

- classical $T\bar{T}$ and $J\bar{T}$ – deformed CFT possess an ∞ # of symmetries → field-dependent coord. transf.

MG, Monten '20

$$Q_f = \int d\sigma f(u) \mathcal{H}_L \qquad \bar{Q}_{\bar{f}} = \int d\sigma \bar{f}(v) \mathcal{H}_R \qquad \forall\, f(u), \bar{f}(v)$$

$$\partial_t Q - \{H, Q\} = 0$$

+ affine U(1) in $J\bar{T}$

- u, v → field -dependent coordinates → universal for each deformation

$$T\bar{T} \;:\quad u \sim \sigma + t + 2\mu \int d\sigma\, \mathcal{H}_R \,, \qquad v \sim \sigma - t + 2\mu \int d\sigma\, \mathcal{H}_L$$

$$J\bar{T} \;:\quad u = \sigma + t \,, \qquad v \sim \sigma - t - \lambda \phi \quad \leftarrow \text{bosonisation of } J \qquad J = \star d\phi$$

- in terms of u, v (classical) $T\bar{T}$/$J\bar{T}$ dynamics → original CFT

- symmetry algebra: 2 commuting copies of Witt or Witt-KM algebra

"original CFT symmetries seen through the prism of the dynamical coordinates"

Classical J$\bar{\text{T}}$ symmetries on compact space

$$u = \sigma + t \,, \qquad v \sim \sigma - t - \lambda\phi$$

- compact space → v zero mode has P.B. that are inconsistent w/ charge & momentum quantization
- can remove zero mode while keeping charge conserved: $v \to v_{imp} \,, \quad \bar{Q} \to \bar{\mathcal{Q}} \,, \quad \bar{J} \to \bar{\mathcal{J}}$
- this affects RM charge algebra → non-linear modification of Witt-KM
- however, the nonlinear combinations

flow equation w.r.t. $\mathcal{X}^{cls}_{J\bar{T}}$

$$\begin{cases} \tilde{L}^{\lambda,cls}_n = R\,Q_n - \lambda H_R J_n + \dfrac{\lambda^2 H_R^2}{4}\delta_{n,0} \,, & \tilde{J}^{\lambda,cls}_n = J_n - \dfrac{\lambda H_R}{2}\delta_{n,0} \\ \tilde{\bar{L}}^{\lambda,cls}_n = R_v \bar{\mathcal{Q}}_n - \lambda H_R \bar{\mathcal{J}}_n + \dfrac{\lambda^2 H_R^2}{4}\delta_{n,0} \,, & \tilde{\bar{\mathcal{J}}}^{\lambda,cls}_n = \bar{\mathcal{J}}_n - \dfrac{\lambda H_R}{2}\delta_{n,0} \end{cases}$$

→ non-linear relation between H and $\tilde{L}^{\lambda}_0, \tilde{\bar{L}}^{\lambda}_0$

- T$\bar{\text{T}}$ similar: w. i. p with R. Monten, I. Tsiares

Defining "primary" operators

- main idea: use interplay of the two sets of symmetry generators

$$\begin{cases} \tilde{L}_n^\mu = R\, L_n - \lambda H_R J_n + \frac{\lambda^2 H_R^2}{4}\delta_{n,0}\,, \qquad \tilde{J}_n^\mu = J_n - \frac{\lambda H_R}{2}\delta_{n,0} \\ \tilde{\bar{L}}_n^\mu = R_v \bar{L}_n - \lambda : H_R \bar{J}_n : + \frac{\lambda^2 H_R^2}{4}\delta_{n,0}\,, \qquad \tilde{\bar{J}}_n^\mu = \bar{J}_n - \frac{\lambda H_R}{2}\delta_{n,0} \end{cases}$$

assumed

full quantum

- introduce auxiliary operators $\tilde{\mathcal{O}}(w,\bar{w})$: $\partial_\lambda \tilde{\mathcal{O}}(w,\bar{w}) \equiv [\mathcal{X}_{J\bar{T}}, \tilde{\mathcal{O}}(w,\bar{w})]$ unphysical

  → identical correlation f. and Ward identities w.r.t. $\tilde{L}_n$ etc., as the ops. in the undeformed CFT

- LM: operators should be primary w.r.t. L_n, J_n → primary Ward identities w/ $h = \tilde{h} + \lambda\bar{p}\tilde{q} + \frac{\lambda^2\bar{p}^2}{4}$

- RM: momentum space $\bar{p}$, primary condition w.r.t. $\bar{L}_n$??? → guess !

$$\boxed{\mathcal{O}(p,\bar{p}) = \int dw d\bar{w}\, e^{-pw-\bar{p}\bar{w}} e^{Aw+B\bar{w}} e^{\lambda\bar{p}\sum_{n=1}^{\infty}(e^{nw}\tilde{J}_{-n}+e^{n\bar{w}}\tilde{\bar{J}}_{-n})}\tilde{\mathcal{O}}(w,\bar{w}) e^{-\lambda\bar{p}\sum_{n=1}^{\infty}(e^{-nw}\tilde{J}_n+e^{-n\bar{w}}\tilde{\bar{J}}_n)}}$$

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Correlation functions

$$\mathcal{O}(p,\bar{p}) = \int dw d\bar{w}\, e^{-pw-\bar{p}\bar{w}} e^{Aw+B\bar{w}} e^{\lambda\bar{p}\sum_{n=1}^{\infty}(e^{nw}\tilde{J}_{-n}+e^{n\bar{w}}\tilde{\bar{J}}_{-n})}\tilde{\mathcal{O}}(w,\bar{w})e^{-\lambda\bar{p}\sum_{n=1}^{\infty}(e^{-nw}\tilde{J}_{n}+e^{-n\bar{w}}\tilde{\bar{J}}_{n})}$$

- Ward identities w.r.t $\bar{L}_n, \bar{J}_n$ → CFT$_2$ Ward identities in the decompactification limit $R \to \infty$

$$h = \tilde{h} + \lambda\bar{p}\tilde{q} + \frac{\lambda^2\bar{p}^2}{4} \qquad \bar{h} = \tilde{\bar{h}} + \lambda\bar{p}\tilde{\bar{q}} + \frac{\lambda^2\bar{p}^2}{4}$$

- arbitrary correlation functions → $\tilde{\mathcal{O}}$ correlators = undeformed CFT correlators in flowed vacuum

$\Rightarrow$ all correlation functions of $\mathcal{O}(p,\bar{p})$ are entirely determined by original CFT correlators

- e.g., 2 & 3 – point functions = CFT momentum-space correlators, but with $\tilde{h} \to h(\bar{p})$, $\tilde{\bar{h}} \to \bar{h}(\bar{p})$

same behaviour as seen in black holes !!!

Summary

- concrete examples of non-local CFT$_2$s exist

- their properties reproduce expectations from the Kerr/CFT correspondence (Virasoro & h(p))

  → explicit mechanism that reconciles the Virasoro symmetries with the non-locality

  → correlation functions: can be defined

  - entirely fixed in terms of those of the undeformed CFT

Future directions:

- most general non-local CFTs?

  → observables & their properties?

  → axiomatic definition? bootstrap?

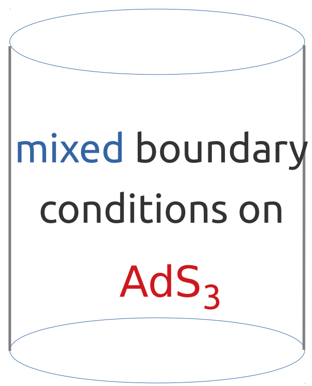
- holography for extremal black holes?

Thank you !

Lessons for holography

$T\bar{T}$, $J\bar{T}$: **double-trace**

- universal , $\forall$ large c CFT



Bzowski, M.G. '18
M.G., Monten '19

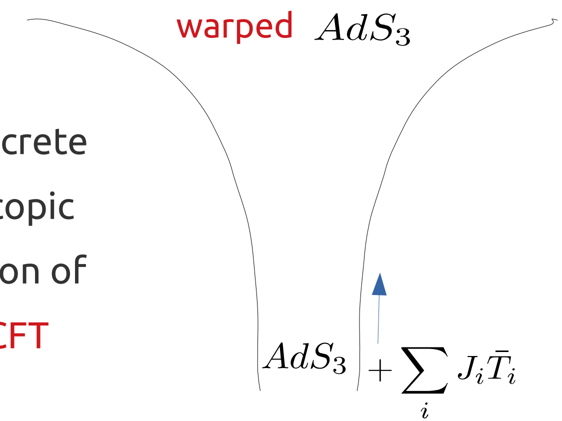
- bulk & boundary th. are independently defined

- currently ASG analyses do not reproduce boundary symmetries → change rules ?

Single-trace $T\bar{T}$, $J\bar{T}$ deformation $\sum_{i=1}^{p} J_i \bar{T}_i$

- near horizon NS5-F1 → $\mathcal{M}^p / S_p$

first concrete
microscopic
realisation of
Kerr/CFT



Apolo, Song
Chakraborty, Giveon, Kutasov

- Virasoro symmetries survive s.p. orbifold

w.i.p. Chakraborty & Georgescu

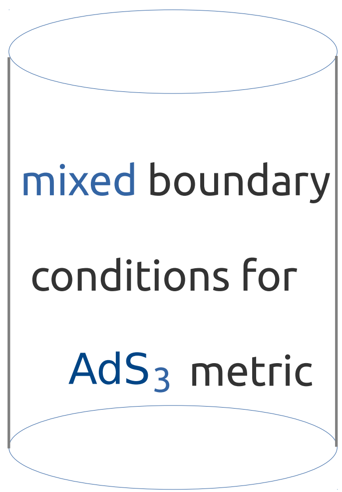
- detailed holographic dictionary?

Generalisations?

Holography for $T\bar{T}$, $J\bar{T}$ - deformed CFTs

$T\bar{T}$, $J\bar{T}$ deformations : **double trace**

- universal , $\forall$ large c CFT



precision holography : perfect match of bulk/ boundary spectrum

- "top-down": boundary theory known

  → derive properties of bulk gravity th.

- contrast with usual "bottom-up" approaches:

  consistent-looking bulk theory (full set??)

  → infer properties of boundary theory

- symmetry generators in finite size must have the zero mode of the f-dep. coord. Removed

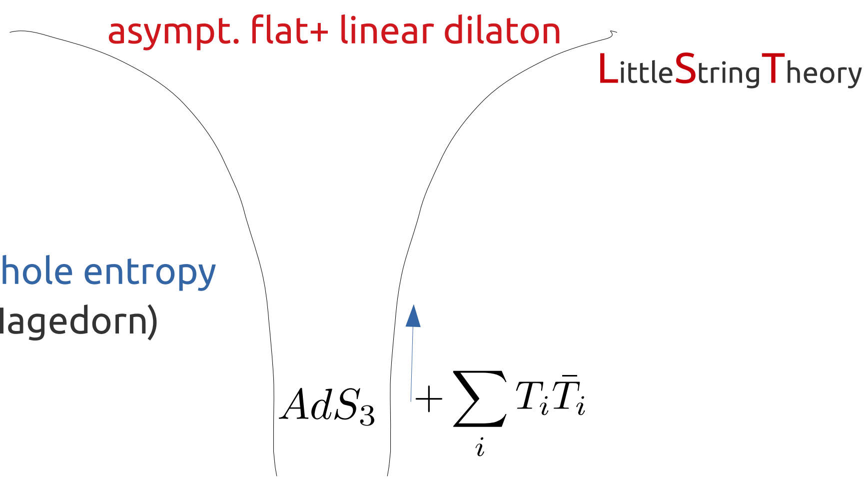
  → looks unnatural from bulk point of view

New rules for asymptotic symmetries?

Single-trace $T\bar{T}$, $J\bar{T}$ deformation

Single-trace $T\bar{T}$ deformation $\sum_{i=1}^{p} T_i \bar{T}_i$

- near horizon NS5-F1 → $\mathcal{M}^p / S_p$



✓ black hole entropy (Hagedorn)

Giveon, Itzhaki, Kutasov

- analogous single-trace $J\bar{T}$ deformation

Apolo, Song; Chakraborty, Giveon, Kutasov

- concrete holographic duals to non-AdS spacetimes
- first concrete microscopic proposal for an extremal black hole
- Virasoro symmetries & their bulk realisation?

Generalisations?

The JT̄ holographic dictionary

MG, Bzowski '18

- introduce sources: $J^\alpha \leftrightarrow \mathrm{a}_\alpha \qquad T^a{}_\alpha \leftrightarrow e^a{}_\alpha$
- variational principle:

$$\delta S_\mu = \delta S_{CFT} - \delta S_{J\bar T} = \int d^2x \, [\overbrace{eT^a{}_\alpha \delta e^\alpha_a + eJ^\alpha \delta \mathrm{a}_\alpha}^{\text{CFT}} - \delta(\overbrace{\mu_a T^a{}_\alpha J^\alpha e}^{\text{deformation}})] = \int d^2x \tilde e (\overbrace{\tilde T^a{}_\alpha \delta \tilde e^\alpha_a + \tilde J^\alpha \delta \tilde{\mathrm{a}}_\alpha}^{\text{new sources \& vevs}})$$

- new sources $\qquad \tilde e^\alpha_a = e^\alpha_a - \mu_a \langle J^\alpha \rangle \,, \qquad \tilde{\mathrm{a}}_\alpha = \mathrm{a}_\alpha - \mu_a \langle T^a_\alpha \rangle$
- new vevs $\qquad \tilde T^a{}_\alpha = T^a{}_\alpha + (e^a_\alpha + \mu_\alpha J^a)\, \mu_b T^b{}_\beta J^\beta \,, \qquad \tilde J^\alpha = J^\alpha$

large N field theory

Holography:

$(T^a{}_\alpha, e^a{}_\alpha)$ modelled by 3d Einstein gravity

$(J^\alpha, \mathrm{a}_\alpha)$ $U(1)$ Chern-Simons gauge field

non-dynamical

- AdS_3 gravity with mixed boundary conditions (CSS-like, but allowing full dynamics)
- perfect match between energies of black holes and the defomed CFT spectrum ✔
- asymptotic symmetry group:

$$\underbrace{SL(2,\mathbb{R})_L \times U(1)_L}_{\text{Virasoro - Kac-Moody}} \times U(1)_R$$

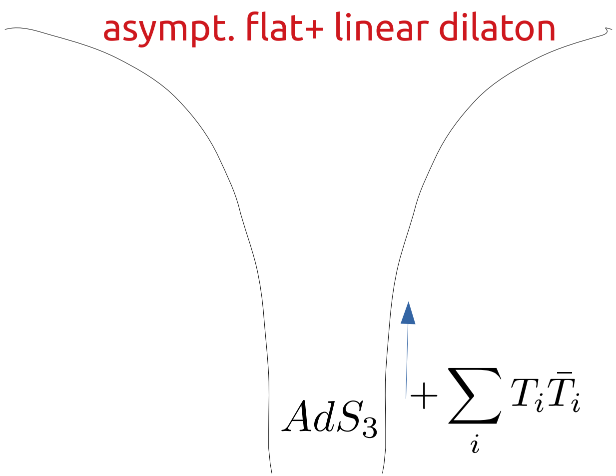
J

non-local

Virasoro - Kac-Moody $\times$ Virasoro$_R$ $\leftarrow f(x^- - \lambda \int J)$

non-local CFT!

The NS5 - F1 system



N_5 NS5 and N_1 F1 strings in the NS5 decoupling limit

$$g_s \to 0\ , \quad \alpha' \qquad \text{fixed}$$

UV: Little String Theory

non-gravitational, non-local theory with Hagedorn growth

IR: AdS_3 dual to $(\mathcal{M}_{6N_5})^{N_1}/S_{N_1}$ symmetric orbifold CFT

- can be obtained via TsT of near norizon AdS

- worldsheet σ - model : exactly marginal deformation of the $SL(2,\mathbb{R}) \times SU(2) \times U(1)^4$ WZW model

  that describes the near-horizon AdS_3 by $J^-\bar{J}^-$

- expand infinitesimally around IR AdS_3 → source for (2,2) single-trace operator $\sum_i T_i\bar{T}_i$

The primary condition

- main idea: use interplay of the two sets of symmetry generators

$$\begin{cases} \tilde{L}^{\mu}_n = R\, L_n - \lambda H_R J_n + \dfrac{\lambda^2 H_R^2}{4}\delta_{n,0}\,, \qquad \tilde{J}^{\mu}_n = J_n - \dfrac{\lambda H_R}{2}\delta_{n,0} \\ \tilde{\bar{L}}^{\mu}_n = R_v \bar{L}_n - \lambda : H_R \bar{J}_n : + \dfrac{\lambda^2 H_R^2}{4}\delta_{n,0}\,, \qquad \tilde{\bar{J}}^{\mu}_n = \bar{J}_n - \dfrac{\lambda H_R}{2}\delta_{n,0} \end{cases}$$

assumed

full quantum

- algebra LM (L_n, J_n) : Virasoro-Kac-Moody ; algebra RM $(\bar{L}_n, \bar{J}_n)$: non-linear modification of Vir.-KM

- LM: operators should be primary w.r.t. L_n, J_n ← implement conformal & affine U(1) transf.

Ward id: $$[L_n, \mathcal{O}(w)] = e^{nw}(nh\mathcal{O} + \partial_w \mathcal{O})_{n \geq -1} \quad \text{w/} \quad h = \tilde{h} + \lambda \bar{p}\tilde{q} + \frac{\lambda^2 \bar{p}^2}{4}$$

- introduce auxiliary ops. $\tilde{\mathcal{O}}(w,\bar{w})$ defined via $\partial_\lambda \tilde{\mathcal{O}}(w,\bar{w}) = [\mathcal{X}_{J\bar{T}}, \tilde{\mathcal{O}}(w,\bar{w})]$ ← identical correlation functions and Ward identities w.r.t. $\tilde{L}_n$ etc., as the operators in the undeformed CFT

$$\boxed{\mathcal{O}(w,-) = e^{Aw} e^{\lambda\bar{p}\sum_{n=1}^{\infty} e^{nw}\tilde{J}_{-n}}\tilde{\mathcal{O}}(w,-) e^{-\lambda\bar{p}\sum_{n=1}^{\infty} e^{-nw}\tilde{J}_n} \times RM}$$

Proposed holographic duality

$$Z_{string}[\text{NS5- F1}] = Z\left[(T\bar{T} - \text{def. CFT}_{6N_5})^{N_1}/S_{N_1}\right]$$

Giveon, Itzhaki, Kutasov '17

+

- RHS is well-defined at finite deformation
- spectrum of string excitations exactly matches $T\bar{T}$ spectrum
- black hole entropy (Hagedorn)
- correlation functions $\langle O(p)O(-p)\rangle$ using worldsheet

−

- uses free product structure in an essential way
- not clear how to deform away from this (singular) point in moduli space
- naively different behaviour from $T\bar{T}$ correlator

more checks?

- similar story holds for $J\bar{T}$: pure NS-NS string background obtained from $AdS_3 \times S^3 \times T^4$ + TsT on one AdS and one angular direction → warped AdS_3 Apolo, Song '18, Chakraborty et al. '18

field-dependent?

- universal near-horizon geometry of extremal black holes, with Virasoro x Virasoro ASG

Resolution

1. Solution for v determined up to a constant → fix such that charge quantization is respected

$$v_{new} = \sigma - \lambda\phi + \frac{\lambda R_v}{R - \lambda Q_K}\widetilde{\phi}_0 \qquad \widetilde{\phi}_0 = \phi_0 - \frac{\lambda}{R_v}\int d\sigma\hat{\phi}(\mathcal{J}_- + \frac{\lambda}{2}\mathcal{H}_R)$$

generator of spectral flow in $J\bar{T}$

- modified charges $\bar{\mathcal{Q}}_n = \int d\sigma e^{-in v_{new}/R_v}\mathcal{H}_R$ are conserved and have Poisson brackets that are consistent with semiclassical quantization

- new charge algebra has quadratic terms on the RHS

- the combinations
$$\begin{cases} \tilde{Q}_n = R\,Q_n - \lambda E_R K_n + \dfrac{\lambda^2 E_R^2}{4}\delta_{n,0}\,, \quad K_n - \dfrac{\lambda E_R}{2}\delta_{n,0} \\ \tilde{\bar{\mathcal{Q}}}_n = R_v\bar{\mathcal{Q}}_n - \lambda E_R\bar{\mathcal{K}}_n + \dfrac{\lambda^2 E_R^2}{4}\delta_{n,0}\,, \quad \bar{\mathcal{K}}_n - \dfrac{\lambda E_R}{2}\delta_{n,0} \end{cases}$$
do satisfy Witt-Kac-Moody^2

2. $\tilde{Q}_0,\ \tilde{\bar{\mathcal{Q}}}_0$ coincide with the undeformed CFT energies $E^{(0)}_{L,R}$ ← integer-spaced spectrum

→ not the left/right energies in the JT – deformed CFT!