Consistency Conditions for Rapid Turn Inflation

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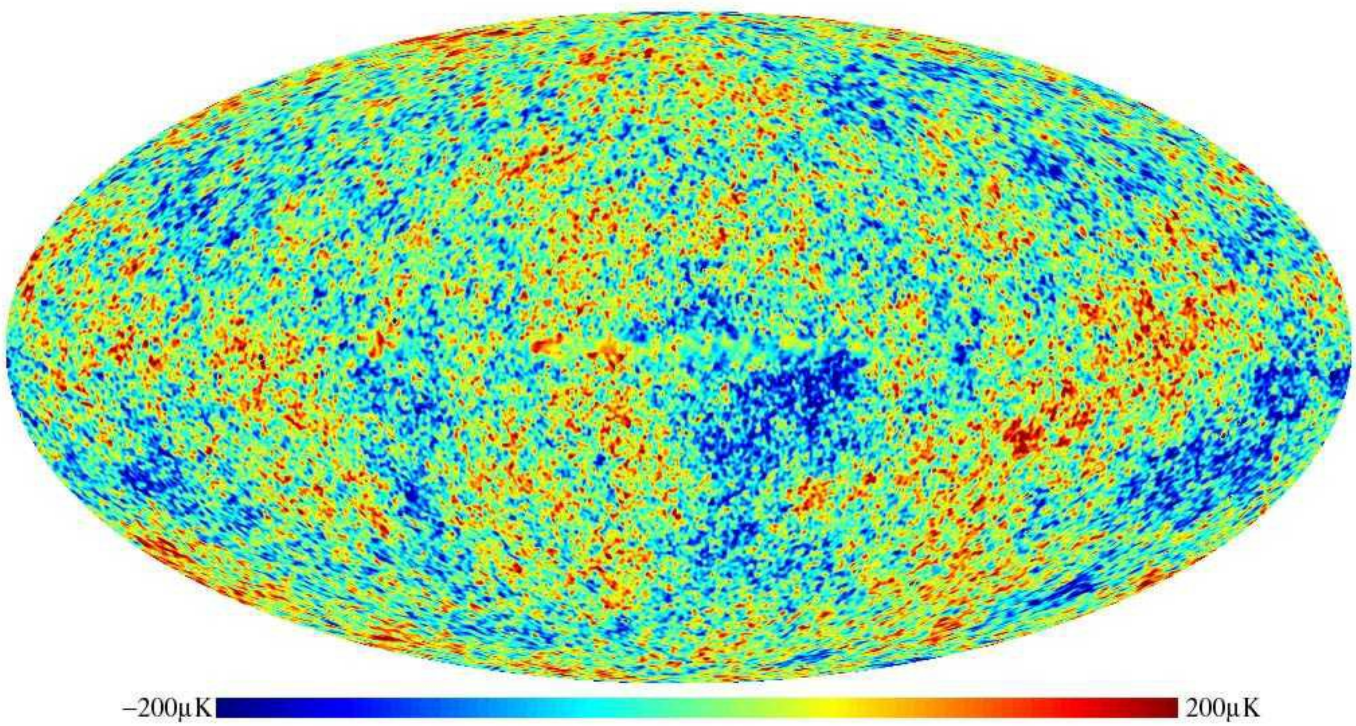
(to appear soon)

Motivation

Cosmic Microwave Background (CMB) radiation:

WMAP and Planck satellites:

Detailed map of CMB temperature fluctuations on the sky



$\overline{T} = 2.7\mathrm{K}$

According to CMB data:

Temperature fluctuations $\dfrac{\delta T(\theta,\varphi)}{\overline{T}}$, (θ,φ) coord. on S^2 ,

measured with great precision:

- On large scales:

  Universe is homogeneous and isotropic

- In Early Universe:

  Small perturbations that seed structure formation

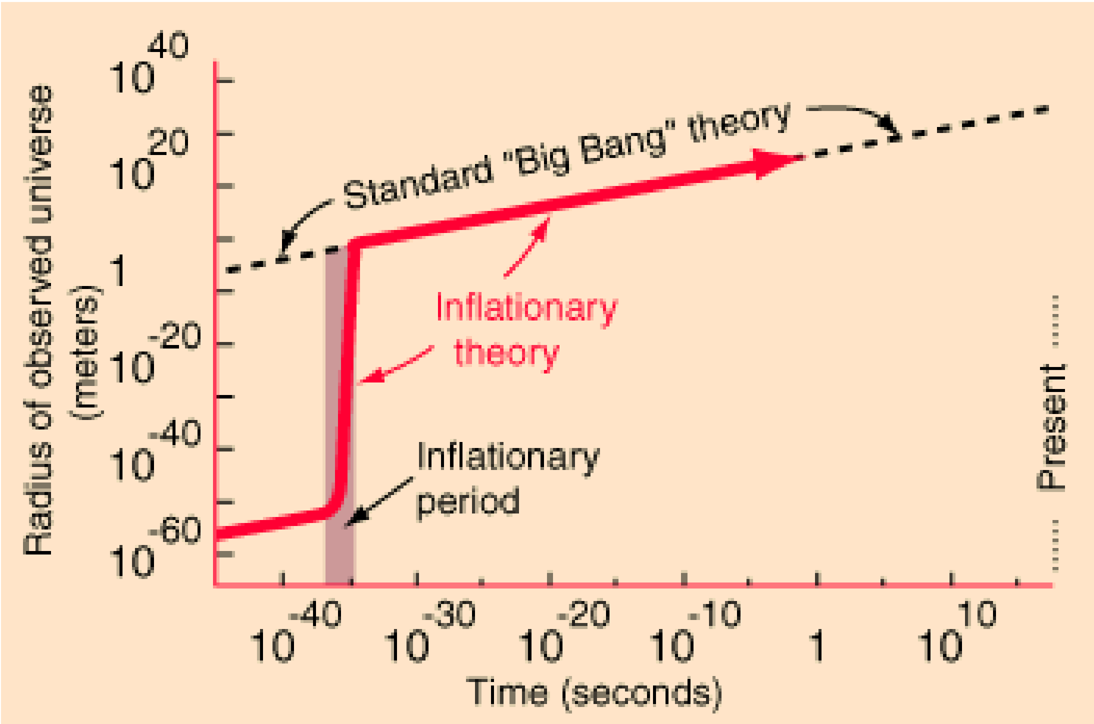
  [(Clusters of) Galaxies]

Cosmological Inflation:

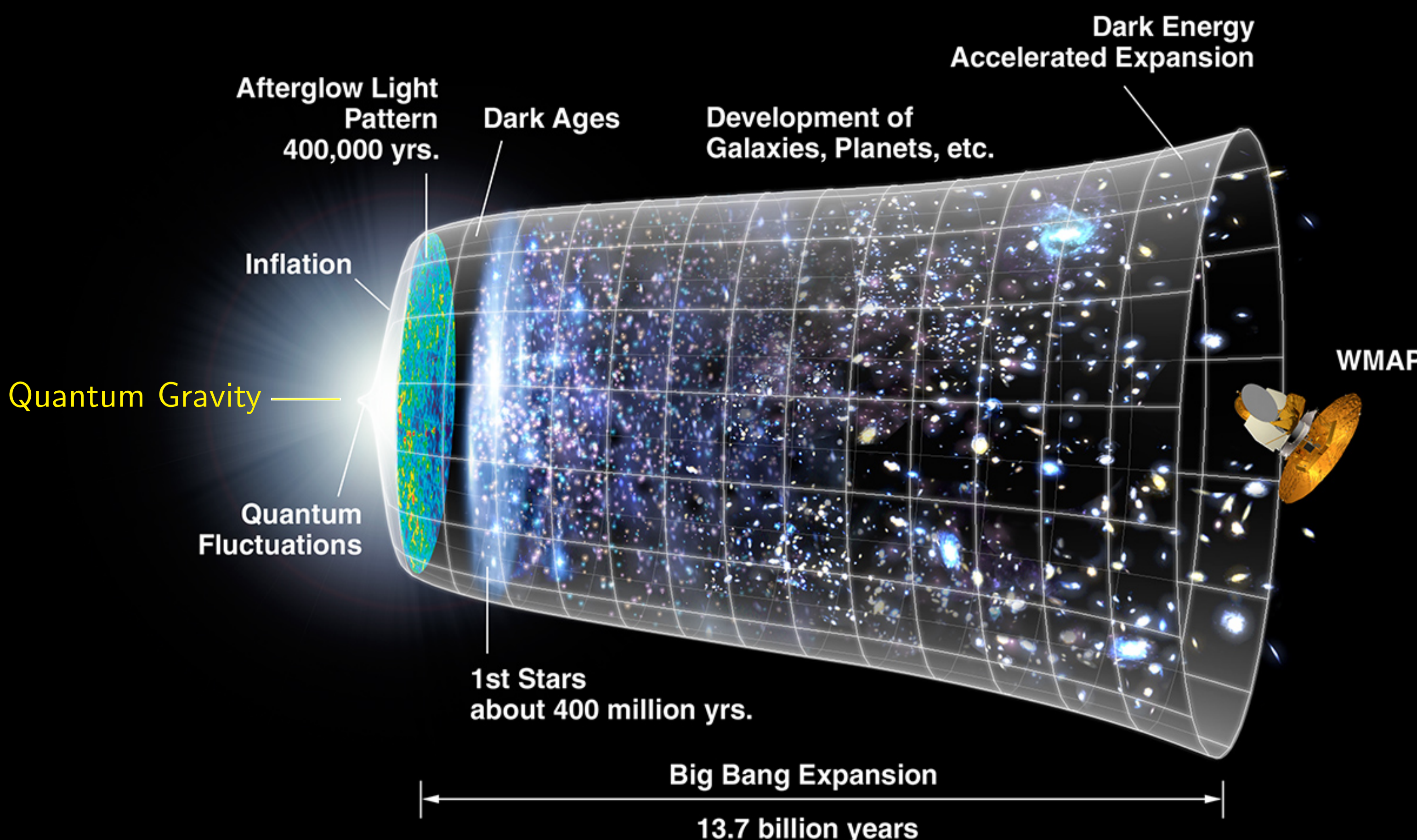
Period of very fast expansion of space in the Early Universe

(faster than speed of light)

$\Rightarrow$ homogeneity and isotropy observed today



Inflation: Traces of Quantum Gravity ?



(Shortly after) Big Bang: Origin of all structure we see today!

NASA/WMAP Science Team

Cosmological Inflation:

Standard description:

– expansion driven by the potential energy of a single scalar field φ called inflaton

– weakly coupled Lagrangian for the inflaton within QFT framework:

$$S = \int d^4x \, \sqrt{-\det g} \left[\frac{R}{2} - \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \, \partial_\nu \varphi - V(\varphi) \right]$$

– slow roll approximation:

$$\epsilon_{\rm v} \overset{\rm def.}{=} \frac{1}{2} \left[\frac{V'(\varphi)}{V(\varphi)} \right]^2 \ll 1 \quad , \quad \eta_{\rm v} \overset{\rm def.}{=} \frac{V''(\varphi)}{V(\varphi)} \ll 1$$

BUT: Many reasons to consider non-standard models

- Embedding in a fundamental theory:
  - In string compactifications 4d scalars arise in pairs (chiral superfields)
  - Compatibility with quantum gravity ('swampland' conjectures, in particular, constraints on $V(\varphi)$; very restrictive for a single scalar)
- Richer phenomenology:
  - Decoupling the generation of curvature perturbations (curvaton) from the inflaton
  - Non-Gaussianity of primordial fluctuations

Two-field Inflationary Models

Action:

$$S = \int d^4x \sqrt{-\det g}\left[\frac{R}{2} - \frac{1}{2} G_{ij}(\varphi)\, g^{\mu\nu}\, \partial_\mu \varphi^i\, \partial_\nu \varphi^j - V(\varphi)\right] ,$$

$g_{\mu\nu}(x)$ - spacetime metric ,

Ansatz: $ds_g^2 = -dt^2 + a(t)^2 d\vec{x}^2$, $\varphi^i = \varphi^i(t)$,

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)} \quad - \quad \text{Hubble parameter} ,$$

$G_{ij}(\varphi)$ - target space metric: $i, j = 1, 2$

In general, curvature of G_{ij} - nonvanishing

Conceptual note:

In single-field models potential $V(\varphi)$ plays key role:

Always: field redefinition $\rightarrow$ canonical kinetic term

(Can transfer complexity to the potential)

In multi-field models:

Cannot redefine away the curvature of G_{ij} !

(I.e., kinetic term becomes important !)

$\Rightarrow$ Can have: - Genuine two (or multi-) field trajectories even when $\partial_{\varphi^i} V = 0$ for some i

- New phenomena due to non-geodesic motion in field space

Characteristics of a background trajectory:

Background trajectory $(\varphi^1(t), \varphi^2(t))$ in field space:

Tangent and normal vectors: $\quad i, j = 1, 2$

$$T^i = \frac{\dot{\varphi}^i}{\dot{\sigma}} \quad , \quad \dot{\sigma}^2 = G_{ij}\dot{\varphi}^i\dot{\varphi}^j$$

$$N_i = (\det G)^{1/2}\epsilon_{ij}T^j$$

(Note: $\quad N_i T^i = 0 \quad , \quad T_i T^i = 1 \quad , \quad N_i N^i = 1$)

Turning rate of the trajectory:

$$\Omega = -N_i D_t T^i \quad ,$$

$$D_t T^i \equiv \dot{\varphi}^j \, \nabla_j T^i = \dot{T}^i + (\Gamma_G)^i_{jk} \, \dot{\varphi}^j \, T^k$$

Characteristics of a background trajectory:

Equivalently, the turning rate:

$$\Omega^2 = G_{ij}(D_t T^i)(D_t T^j) = ||D_t T^i||^2$$

Slow-roll parameters:

$$\varepsilon = -\frac{\dot{H}}{H^2} \quad , \quad \eta^i = -\frac{1}{H\dot{\sigma}} D_t \dot{\varphi}^i \qquad \left[\varepsilon \neq \varepsilon_V = \tfrac{1}{2}\tfrac{G^{ij} V_i V_j}{V^2} \right]$$

Expand: $\eta^i = \eta_{||} T^i + \eta_{\perp} N^i \quad \rightarrow \quad \eta_{||} = -\dfrac{\ddot{\sigma}}{H\dot{\sigma}} \;\; , \;\; \eta_{\perp} = \dfrac{\Omega}{H}$

$\varepsilon\,, \eta_{||}$: same as for single-field inflation with inflaton $\sigma(t)$

Slow roll: $\varepsilon\,, |\eta_{||}| \ll 1$; Rapid turn: $\eta_{\perp}^2 \gg 1$

Rapid Turn Inflation

Pheno viability and perturbative stability:

In the past:

Slow roll & slow turn: $\varepsilon \, , |\eta_{||}| \ll 1 \quad \& \quad \eta_{\perp} \ll 1$

Recently also:

Slow roll & rapid turn: $\varepsilon \, , |\eta_{||}| \ll 1 \quad \& \quad \eta_{\perp}^2 \gg 1$

Pheno interest:

– Rapid turn regime can be realized in steep potentials

– Brief rapid turn, during slow roll, induces PBH generation

Long-term rapid-turning inflationary models?

Full-fledged rapid turn inflation:

To achieve $\sim$50-60 or so efolds of inflation, need to sustain rapid-turning regime for a prolonged period

For sustained rapid turning, require: $\nu \equiv \frac{\dot{\eta}_\perp}{H\eta_\perp} \ll 1$

Various models in field theory:

Hyperinflation, side-tracked inflation, angular inflation, ...

(A. Brown, arXiv:1705.03023 [hep-th]; S. Garcia-Saenz, S. Renaux-Petel, J. Ronayne, arXiv:1804.11279 [astro-ph.CO]; P. Christodoulidis, D. Roest, E. Sfakianakis, arXiv:1803.09841 [hep-th]; A. Achucarro, E. Copeland, O. Iarygina, G. Palma, D. Wang, Y. Welling, arXiv:1901.03657 [astro-ph.CO]; T. Bjorkmo, arXiv:1902.10529 [hep-th]; ...)

Recent claim: (V. Aragam, R. Chiovoloni, S. Paban, R. Rosati, I. Zavala, arXiv:2110.05516 [hep-th])

Difficult to realize sustained rapid-turn inflation in SUGRA

Our work: Consistency conditions for rapid turn inflation

Consistency conditions for rapid turn inflation:

(L. Anguelova and C. Lazaroiu, to appear soon...)

Difficulty (in maintaining slow-roll $\&$ rapid-turn regime) not about SUGRA

Instead: $\exists$ consistency conditions relating $V(\varphi)$ and $G_{ij}(\varphi)$

$\rightarrow$ very limited choices of scalar potentials

Origin of consistency conditions:

Requiring compatibility between:

- Equations of motion (scalar field EoMs, Einst. eqs.)
- Conditions for sustained slow roll $\&$ rapid turn

$$(\varepsilon \, , \, |\eta_{\|}| \, , \, \nu \ll 1 \, , \, \eta_{\perp}^2 \gg 1 \, , \, ...)$$

Amazingly: not studied before !

Consistency conditions for rapid turn inflation:

Useful to change basis in field space: $(T^i, N^i) \to (n^i, \tau^i)$,

$$n^k = \frac{G^{kl}V_l}{\sqrt{G^{ij}V_iV_j}} \quad , \quad \tau_i = (\det G)^{1/2}\epsilon_{ij}n^j$$

Basis (T, N) : determined by field-space trajectory

(generally unknown)

Basis (n, τ) : determined by potential V and metric G_{ij}

(given for each concrete model)

Relation: $T = \cos\theta_\varphi \, n + \sin\theta_\varphi \, \tau$, θ_φ - characteristic angle

$$N = -\sin\theta_\varphi \, n + \cos\theta_\varphi \, \tau$$

Consistency conditions for rapid turn inflation:

On solutions of the equations of motion:

$$V_{TT} = H^2 \left(\eta_\perp^2 + 3\varepsilon + 3\eta_{||} - \xi\right) \quad , \quad \xi = \frac{\dddot{\sigma}}{H^2 \dot{\sigma}}$$

$$V_{TN} = H^2 \, \eta_\perp \left(3 - \varepsilon - 2\eta_{||} + \nu\right) \quad ,$$

where $V_{TT} \equiv T^i T^j \nabla_i V_j$, $V_{TN} \equiv T^i N^j \nabla_i V_j$

We showed that above implies:

$$3 V V_{nn}^2 = V_{n\tau}^2 V_{\tau\tau}$$, where $V_{n\tau} \equiv n^i \tau^j \nabla_i V_j$ etc. ,

in the approximations regime of arXiv:2110.05516 [hep-th]

$(\, \varepsilon, |\eta_{||}|, \xi, \nu \ll 1 \, , \, \eta_\perp^2 \gg 1 \,)$

$\rightarrow$ For given G_{ij} : constraint on V !

Consistency conditions for rapid turn inflation:

Bjorkmo's approximation: (unifies prominent rapid turn models)

(T. Bjorkmo, arXiv:1902.10529 [hep-th])

Expand: $\dot{\varphi}^i = v_n n^i + v_\tau \tau^i$; Def.: $f_n \equiv -\frac{\dot{v}_n}{H v_n}$, $f_\tau \equiv -\frac{\dot{v}_\tau}{H v_\tau}$

– Tacitly assumed: $|\eta_{||}| \ll 1 \iff |f_n|\,,|f_\tau| \ll 1$

– Claimed: $\eta_\perp \gg \mathcal{O}(\varepsilon)\,,\, \nu \ll 1 \longrightarrow$ sol. $\exists$ for any V, G_{ij}

Our results: $*)\ \eta_{||} = \cos^2\theta_\varphi f_n + \sin^2\theta_\varphi f_\tau\,;$

$*)$ slow roll: $f_\tau \approx -\frac{9}{\eta_\perp^2} f_n\,;$ $*)$ rapid turn: $\cos^2\theta_\varphi \ll 1$

Hence: Can have $|\eta_{||}| \ll 1$ with $|f_n| \sim \mathcal{O}(1)\ \&\ |f_\tau| \ll 1$

Also: $\exists$ highly nontrivial consist. cond. relating V and G_{ij}

Summary

We found:

There are consistency conditions for compatibility of long-term slow-roll and rapid-turn inflationary regime with all EoMs

Consistency cond.:

Scalar potential $\longleftrightarrow$ scalar field-space metric

$\Rightarrow$ Difficult to find potentials, which are compatible with these conditions

$\rightarrow$ To reconsider the literature on rapid turn inflation...

Thank you!